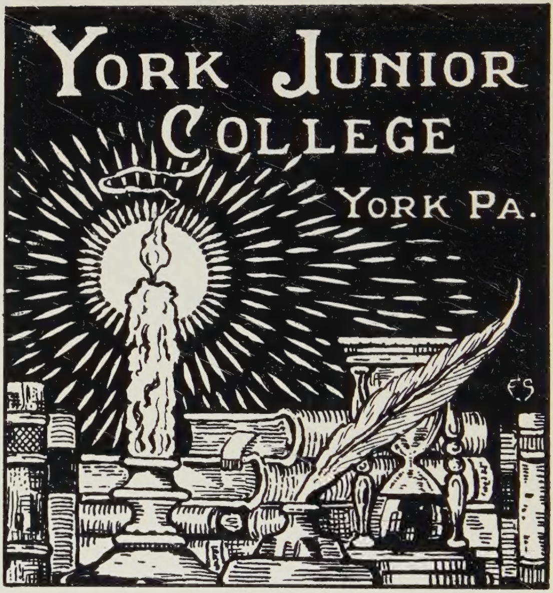

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THE EINSTEIN THEORY
EXPLAINED AND ANALYZED

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THE EINSTEIN THEORY

EXPLAINED AND ANALYZED

BY

SAMUEL H. GUGGENHEIMER

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INTRODUCTION

THE object of this treatise is to explain and analyze the Einstein theory in such a way as, it is hoped, will make it comprehensible to any thoughtful person with a previous foundation of elementary scientific and philosophic reading, approaching that of the serious college graduate; and, in the course of the analysis, to disclose some of the basic assumptions of the theory, and at least to indicate the tests of its soundness. The treatment is philosophic and the mathematical is reduced to a minimum.

It has been remarked, no doubt with considerable exaggeration, that there is not a score of persons in the world who understand the theory. Perhaps the remark was intended to apply only to the mathematical details. If it represents, however, only a wide approximation of the truth, the implied obscurity of the subject must nevertheless cause one to wonder at the number of people who manifest the liveliest interest in it. The reason probably is that the theory appears to be the first scientific attempt to transcend the supposed bounds of human experience and to penetrate the secrets of the hitherto supposed infinite and unknowable, and of what, before Einstein, would have been termed the non-spatial. Science at last appears to venture into the mystic field of metaphysics.

At least since the time of Kant, it has been supposed by most philosophers that the limits of possible knowledge have been fixed. Space and time were supposed to have been proven to be mere forms or conditions of human thought in which, at the most, corresponding modes of external being were apprehended, and not in themselves knowable entities. What a fascination, therefore, for the philosophic mind when a great thinker proposes, on the basis of mathematical demonstrations, to pierce these limits, to substitute for the relative concepts of space and time a continuum of four dimensions, including besides the three we know, time as the fourth, and to establish such apparently mystic propositions as the following: Each body has, and, in a sense, makes its own kind of this space-time continuum. Bodies gain mass with motion. There is no actual or physical, but only relative motion. Its appearance and kind is the effect of the particular pucker or curvature of space-time caused by or perhaps constituting the particular body involved. There is in nature a limiting velocity beyond which bodies cease to be. Gravitation is not an attracting force, but the geometric structure of the particular space-time continuum involved, which compels the appearance of gravitational motion. Such things as length, duration, mass, motion, force, have no actual, but only relative existence, even in the phenomenal world, differing according to the point of view.

This outline of the possible reason for the great interest evinced in the new theory has carried us, however, far beyond mere introductory matter and will be better understood after we have delved deeply into this interesting subject.

The book of Einstein from which quotations are made from time to time is: *Relativity. The Special and General Theory,* by Albert Einstein, Ph.D. Translated by Robert W. Lawson, 1920.

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THE
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THE EINSTEIN THEORY EXPLAINED AND ANALYZED

CHAPTER I

RELATIVITY IN THE PHILOSOPHIC SENSE

THE uninitiated, in reading the simple illustrations of relativity of motion with which Einstein begins his exposition, startling enough to those who have never devoted thought to the subject, quite naturally regard the idea as new. Of course, nothing could be farther from the truth. The idea of the relativity of all knowledge, including motion particularly, is as old as philosophy and probably its chief achievement. In fact, the very beginnings of that all-embracing science, in the early days of the profound and revolutionizing Greek thought, lie in that field. There the penetration of the Greeks more than two thousand years ago is very acute, in spite of their ignorance of the scientific facts which, within the last three centuries, have corroborated so surprisingly their early deductions. The Greeks early recognized that the mind cannot penetrate beyond appearances. Its own states are all it can know. What the appearances or phenomena are in themselves, what, if anything, underlies them, must of necessity lie beyond mortal ken. In that sense knowledge is relative only. We may analyze a complex phenomenon into constituents and deduce from precedent phenomena so-called causes, but in the last analysis, the mind can only know what is present in it, its knowledge is true only relative to itself, and it cannot really know what lies beyond.

When a human being perceives a rose, the Greek argued that, while he may have reason to believe or have faith that there is something outside of his mentality which impresses thereon a certain visual, tactual and olfactory picture, the real thing that creates the perception is manifestly beyond the reach of a mentality which can know nothing beyond its own impressions. The Greek was, therefore, very quick to understand that his knowledge of the rose was really relative in that it must be subject to the reservation that it was strictly confined to the mental picture. What, if anything, existed external to the ego was beyond its reach.

Great as the Greeks were in their deductive reasoning, even in their accumulation of scientific knowledge, slight though the latter was compared with the acquisitions of our own time, the interruption to the development of their thought by the centuries of mediæval turbulence and ignorance deprived them of the intellectual satisfaction of the magnificent corroboration of their deductions in the Newtonian era. With little idea of the true nature of sound and light in the world external to the ego, assumed by science, the Greek well knew that the mental pictures, visual or aural, could not be the reality itself; but in our own era, beginning with Newton, the very means by which the mental sensation or impression is created, were empirically demonstrated.

Whether we accept the undulatory theory of light or not, we now know that vision is produced in animal brains by a motion of some kind, caused by so-called force. We shall be led into no logical error, therefore, if we accept for illustration the undulatory theory, which seems satisfactorily to explain most of light's phenomena. Matter, in its innermost molecular or atomic structure, is set into violent motion by causes not material for the moment, and in turn agitates the ether which is supposed to pervade all space, including the interstices of the atomic structure. When the ethereal undulations or waves or vibrations so caused, trillions of them per second, strike a material object and are thence reflected to the ocular apparatus, the motion of certain of them transmitted through that apparatus and through the molecular particles of the optic nerve and brain is transmuted by the alchemy of that organ into the phenomenon which we know as light. The object is seen.

When rays of light, therefore, from the sun strike, for example, a rose, we now know that the undulations are reflected by the entity which we know as the rose to the intervening eye, and that of course there is no color in the rose but only in the perceiving mind. The rays consisting of waves of a certain length vibrating at the rate, appropriate to that length, of so many trillions per second, are called red, since they cause in us that sensation. Those of a shorter length, vibrating thus with a greater rapidity, are called violet since they cause in us the sensation, violet, and so on. The composite of all the different rates of vibration is known as white light. Waves of certain lengths penetrate into some textures and lose their force in the creation of heat or otherwise. Those of such lengths as may not penetrate the particular texture are reflected from its surface to the observer. The red rose is of such a texture that all the waves are absorbed into it except those of such qualities of length and rate of vibration as produce the mental impression or sensation of redness. These rays are reflected from the rose to the eye, and thence transfer their motion through the optic nerve to the cellular recesses of the brain where that particular class of motion causes that particular sensation. There is, of course, no conceivable identity of the sensation with the vibrations in ether, nerve or brain. The sensation may be said to be the mental product or interpretation of a particular kind of physical motion, in itself one of the deepest mysteries of nature, which we shall later examine.

Even in the time of Locke, it was understood that color is a secondary and not inherent quality of matter, and hence is in the perceiving mind and not in the object. It was reserved for the later thinkers, and particularly for the profound Hume, to realize that no perceived qualities of the object are inherent, all being forms of mental susceptibility, mental phenomena, the underlying reality or cause of which the mind can never truly know.

The odor of the rose, of course, is easily analyzable as a secondary quality in the Locke sense, the mental impression of fragrance being an entirely different thing from the corpuscular exhalations which cause it. The rose, however, in common with all matter, has also a certain resistance to touch, what we designate as *substance.* We touch it and we say there is *something there.* Likewise, to the senses of touch and muscular strain involved in the travel of the fingers over its surface, and to the associated sense of vision, it appears to have the extension common to all substance. Locke did not see, although the later thinkers did, and very clearly, that the impression of a resisting force and of substance extended in space, the "something-there-ness," is entirely mental, being the form into which the crucible of the mind transmutes the force that is apparently acting externally to the thinker, but which in itself he can never know. If a man apply with his fingers pressure to a table, his mind apprehends that there is something there that gives him a certain impression of resistance, but all that he knows is this impression of a particular quality and degree.

If a nurse were to hold an orange before the eye of a new born infant, it is probably true to say that the infant would have little or no impression of the phenomenon, except perhaps the vaguest idea of change from the previous state of its absence; and the mental condition would probably contain no consciousness of externality, or even of the perceiving ego. If we were to imagine the orange repeatedly moved away and restored to the line of vision, we could properly consider that after a certain length of time the change would be noticed; and even more so if the experiment included now and then the substitution of a red apple for a yellow orange. Basing our imagination of the infant psychology on what we know and remember of our own, it seems reasonable to say that the first impression that the infant would have would be of difference. He would realize that there was something different between the red and the yellow, between one or the other being there, and then not being there, but certainly it would be absurd to suppose at this early stage of his development that he would have any conception whatever that the phenomenon was taking place outside of his own mentality.

After a while, however, as his experience increased, he would discover that he had a will belonging to his own ego and controlling his own members, and that this will had nothing to do with the presence or absence of the apple or orange. He would become conscious of his own muscular movements, and he might perhaps on some occasion note, when he felt certain movements in his arms, which his experience would sooner or later discover belonged to his own ego and will, as distinguished from external egos and other external phenomena, that those muscular movements resulted in contacts with external objects other than his own parts. After many infantile experiences, he would in time discover such a contact of his hand when the visual appearance of the yellow orange was present, but which would be lacking when that visual appearance was absent; and he would finally come to associate the impression of contact and of roundness that he had when his hand made the circumference of the orange, with the visual impression. He would also find that the visual impression of roundness and yellowness and the olfactory impression of the orange odor coincided with a certain tactual impression on grasping the orange, while different impressions would be associated with the apple contact.

It is needless to say that the space concept he would finally form would be that caused by muscular motion and later associated with certain qualities of perspective in vision. When he discovered that he could stretch his arm, or, later on, that he could drag himself along the floor, in order to reach and touch the things that his eye saw, he would have received his first experiential knowledge of externality and of space, which would never leave him in all his after life. His conception of an external universe would be the product of the association or complex of innumerable sense impressions and memories; and it would be his invariable experience that his mind could have no conception of objects, of physical motion, of succession, in other words, of things and events, except in terms of time and space.

Nevertheless, it requires very little analysis to see that time and space can be for us nothing in themselves. They are not things that one can see or touch or perceive by any sense. Space is the form of the mental reaction to phenomena of extension and of mass motion, of matter's change of position. Time, without refining too much at this stage, is the form of the mental reaction to sequence in the changes of mass position, perhaps also to molecular, cerebral motion.

The mere fact of existence is the fact of eternity. There is no conceivable time except in the animal brain in its transmutations of some mode of being involved in the mystic phenomena of motion. The universe exists for us only so far as mind extends, consisting only of subject and object, mind and so-called matter, the ego which apprehends and the phenomena extrinsic to and apprehended by it, but which it can only know relatively, that is, in the form in which it apprehends them. The objective is necessary to it. Without something to apprehend there can be no apprehension, no ego; without a consciousness there can be nothing to apprehend. Given subject and object, the wonder is there, unchanging and unchangeable, eternal; and time and space are but forms of finite apprehension of inconceivable realities. Given subject and object, ego and phenomenon, there can be no such thing as nothing. Whatever is, is, but known only as it *appears* to that which knows. The appearances endlessly change, but the persistent and mystic real is eternal.

A so-called red rose, in an absolutely dark room, is, of course, not red. It has no color whatever. A stone is not a stone unless there be a consciousness to feel its resistance or apprehend its visual or other sensuous appearance. A tree falling in a forest makes no noise unless there be a consciousness to apprehend the noise. These propositions cannot be disputed where the terminology is fixed.

The phenomenon of the falling tree is perhaps the best illustration. The sound, of course, is caused by vibrations of air, of a very slow nature indeed as compared with the inconceivably rapid, ethereal vibrations known as light. When a mass vibrates, it sets in vibration the air about it. These vibrations travel until they strike the drum of an animal ear. The drum in turn vibrates and causes the vibrations to be repeated in the aural nerve, whence they finally reach the brain. The brain transmutes these physical motions into the mental sensation of sound. If we understand, therefore, by sound, not the physical vibrations of air, but the mental impression known as sound, there is, of course, no sound in the absence of an animal consciousness. The most we can say is that in the absence of such a consciousness there are vibrations of air; but certainly there is no conceivable identity between the mental impression, "sound" and air motions. We will not carry this analysis to the possible but more recondite conclusion that in the absence of consciousness there is really nothing, further than to suggest the following consideration.

Suppose that everything in the world were to disappear, would it be possible to conceive of the supposed resulting emptiness as an entity? And in the absence of a mind to conceive at all, could there then really be anything? Place in this black emptiness a single object. Has it any existence except in the mind that placed it there, but which under the supposition does not exist? Add to the existence of this single object a disembodied, perceiving mind having in some mystic way the powers, if not the members, of our perceiving organism. Could it perceive anything but the single object, or form any conception of space outside that occupied by the object itself, or of time, in the absence of motion of the object? And could it perceive such motion in the absence of some other body relative to which the change of position would be perceptible?

We have several times mentioned mass motion, which is that class of motion which is apprehended by an animal consciousness in the form of weight, pressure, ocular and other muscular strain, and the like, but our attention has been directed chiefly to those incredibly minute and rapid motions which take place in the molecular, atomic and electronic depths of matter and are apprehended in more subtle, though not more wonderful, forms, such as those of light, heat, sound and the like. Perhaps we have sufficiently treated this branch of the subject in our analysis of sound and vision. One more illustration, however, may not be amiss. When a hand is thrust into hot water, as every educated person now understands, the heat is not in the water, but in the mind. What is in the water is certain motional phenomena. It is unimportant, for our purposes, whether the heat be caused by molecular vibrations or by more profound electrical phenomena. The fact is that some physical motion is taking place in the water, resulting in vibrations of some kind, which are carried along the nerves until they reach the brain, where they are apprehended mentally as the sensation, heat.

The phenomenon of motion is, perhaps, to most philosophic minds, the most mysterious in nature, although all phenomena prove, on analysis, equally wondrous and incomprehensible, reducing themselves in fact to motion. It may be of interest to treat this subject, for a moment, in relation to mass motion. Perhaps the reader has seen a cue strike a billiard ball a thousand times without stopping to reflect and wonder at the strange result. A man contracts or elongates the muscles of his arm by an exercise of the mental phenomenon, intrinsically unknowable, named will. Answering to the exertion of that will the cue strikes. So far as we know, or at least so far as we have known before Einstein, the ball has had nothing added to it, no material physical change has taken place in it, it weighs neither more nor less than it did before, but, answering to that mystic touch, it moves. In other words, will has transferred from the body to the ball something, the nature of which is absolutely unknown to us, but which we call energy. In the absence of the friction of the billiard table and of the gravitational pull of the earth and of any other physical interference, we know that the ball would fly out into space and travel forever in the direction given it by the motion of the cue.

Since the time of Newton, we know that any mass hurled into space, uninterfered with by the gravitational pull of another mass or other interfering force, will travel forever in its initial direction. We know that this motion energy is indestructible. We know, for example, that, if the moving mass be interfered with by collision, the energy of mass motion will be converted into the energy of molecular heat, reaching, under given conditions, to chemical disintegration. But bear in mind that our knowledge of motion is relative only. Molar motion means nothing to us but the change in position of one body in relation to another. All these energies of motion, mass or molecular, can only be apprehended by us as perception of such change of position or in the form of such impressions as light or heat or pressure or violent shock, or the like. So also with force. We know that when we lift a heavy weight there is a pull of resistance, but that which apprehends the pull as resistance is certainly not in the lifted body but in us. We believe there is something external to us which causes the mental sensation, but that something we can never apprehend because it is not within us.

When the scientist, working empirically, traces phenomena through their antecedents from cause to cause, it has been usual to argue that he must ultimately reach a unitary first cause, beyond which he cannot go. That first cause must be infinite since, if it be not, there must be something beyond it, which implies another first cause; and it must be absolute, since if it were relative to something else, there must exist that something else beyond it, in which case it could neither be the first cause nor infinite. Moreover, it must be self-existent, since otherwise it could not be the first cause but must be the product of a prior one. Thus, whenever we believe ourselves to be making some sort of approach to the reality of things, we never fail to discover the restricted nature of our mental apparatus in the fact that the final principles we reach prove to be inconceivable to us, not only because they are never present in our minds except in limited and phenomenal form, but because they must be of a nature that transcends our experience and our finite powers of conception.

In every way, then, it becomes clear that all our knowledge is relative only. If the limits and requirements of this treatise permitted, it would be easy to show that we even think only in terms of relations and that our logical processes are a development from perceptions of relations, such as likeness and unlikeness. It will suffice for our purpose, however, that we have shown the relativity of our knowledge in reference to our elemental perceptions and concepts and in reference to the limits of mental experience and power.

When we have sensations of vision or sound or taste or smell or touch or heat or muscular strain, it is the sensations only that we know. If we choose, as we must, to conceive of them as indicative of an external world, and from them derive the concepts necessary to our existence in that world, those of space and time and matter and motion and cause, we find again that in the nature of things these concepts, being mental only, can be in nowise logically expressive of true external realities. If we then make the assumption, not only of common life, but of science, that these elemental impressions and developed concepts of an external world in time and space are true, and pursue our scientific investigations on that basis, we find once more an inductive and striking proof of this general relativity in the fact that science has been actually successful in showing that that in the assumed external world which produces our elemental sensations is of entirely unrelated character, these sensations being our mental reaction to various phenomena of force and motion. When finally we attempt to reach some idea of the ultimate nature and origin of things as a whole, the very sublimity and unearthliness of the conceptions we are surprisingly able to attain remind us that our powers are, not only in strength, but in kind, unsuited for the task.

In other words, what we have been trying to elucidate is that we are living in a world of the most profound mystery. Its inner meaning is beyond human understanding. Certain of its phenomena and their relations to us and to each other as perceived by us, are indeed revealed, but their respective underlying realities, termed by the philosophers noumena, are forever inscrutable. Thus has it been decided by logic until now deemed conclusive.

It must be understood that, in the analysis of this chapter, the endeavor has been to state the facts upon which alone indubitable knowledge is predicated, without prejudice to such metaphysical interpretation as may suit individual taste. It is, of course, true that many minds cannot and will not remain content with the limitations imposed by these facts. If anyone assumes to know the nature of the underlying reality, of the existence of which most of us have an abiding conviction, as, for example, that this reality itself has consciousness or some nature of that order which transcends our poor notions, or that this underlying consciousness is at one with our own, no one can deny the possibility of the truth of these conceptions, but they are beyond the domain of science. They lie within the domain of speculative metaphysics or of theology.

A mind may perceive splendid combinations of color or sweet harmonies of sound. Inductive analysis may show these to be due to cerebral, molecular motion caused ultimately by physical vibrations in an external medium, but this precludes no one from denying that motion represents their true nature. It precludes no one from imagining that the vibrations themselves may be the expression of an infinite and absolute consciousness in which they may have all the aspects of heavenly symphonies and unimaginable glories of color. Science is no enemy of the poet or metaphysician or reverent devotee who feels the pulse of divine power throbbing behind the manifestations which alone we can know. But let there be no confusion between the realm of science and that of feeling or speculation.

This being clear, it may be conceded that, as above indicated, there is a certain sensitive type of intellect, perhaps the highest type we have, which has never remained happy within the supposed limits and has always felt or hoped, apart from the claims of supernatural revelation made by outgrown creeds, themselves born in the childhood of the race from the speculations of the same type of intellect, that the human mind may be able, in some way as yet unknown to logic, to penetrate further into the riddle of the universe. We moderns, no more than the Greeks, such is the ambition of men, have been able to refrain from indulging this hope, however much, more than they, we have believed ourselves to have fixed the limits of possible knowledge. And it may be further conceded in favor of that hope, that the human mind, in its noblest efforts, constantly reaches sublime heights where it seems to tremble upon the verge of the unseen world.

We know that we can never know the infinite since it is but a vague idea beyond animal experience, and yet we have some kind of reverent apprehension, if not of its nature, at least of its existence. We know that while, according to our forms of thought and our experience, we cannot conceive the infinite, neither can we conceive of limits. We are unable to understand how phenomena or space or time or the potentialities which they express can be endless, and yet we are unable to conceive an end to them. We cannot conceive how anything can exist uncaused or unpreceded, and yet we have a dim, but certain, comprehension of the fact that the universe's underlying being, in its entirety and apart from its ceaseless changes in form, is uncaused and self-existent. And the very strength of our perception that our knowledge of things is relative persuades us of the existence, though inaccessible to us, of the absolute of which they are the relative reflection.

Whatever may be thought by the practical, the more sensitive mentalities realize the beauty and sublimity of this ambitious thinking, giving, as it does, some hope to man of his own spiritual importance. It is for that reason, perhaps, that any intellectual effort that seems even to approach a penetration, if not a solution, of the deep mysteries of time and space and matter and energy and motion and consciousness and existence, immediately commands attention. Einstein's theory, however scientific and mathematical may be its form and method, and however physical the form of its alleged probative discoveries, we shall see, on analysis, is probably metaphysical, and in attempting to penetrate beyond the limits of experience holds out to this type of thinker a hope of penetration into ultimates. That is why the theory has attracted such eager, general interest.

The scientist assumes, as he must, the reality of the physical universe, and hitherto he has abandoned to the theologian and metaphysician the investigation of the underlying noumena. Einstein approaches his problem as a physicist and mathematician. The suggestion is hazarded, however, that it is the metaphysical and not the physical in his thought which will be deemed by many the important end; and that even if his theory, supposedly purely scientific, should prove on analysis unsound, his efforts will prove a spur to rebellion against the ramparts of the hitherto supposedly unknowable. He may endeavor to prove scientifically attributes of space entirely inconceivable, but, if he fail, what will still interest men in his effort is that the subject should have been studied at all. What they really want to penetrate is the mode of being of which space is the mental form.

Philosophy is the unifying science which attempts to generalize all the sciences, and, so far as possible, to reach a conception of the universe and its underlying reality. Philosophy has also been known as metaphysics, and particularly so from the standpoint of its early days, when its investigations favored specially the noumenal world. The origin of the term seems to have been that in Aristotle's works, his writings on physics were followed by those on philosophy, which was therefore called metaphysics, because the Greek word "meta" means "after." That word was also, however, deemed to mean "beyond," and thus the name became an excellent one for that branch of human inquiry which was directed not only to a general unification of knowledge but toward the solution of those things that lie beyond the physical. However, after philosophy deemed itself to have imposed logical limitations to possible human knowledge, perhaps the only scientifically valid branch of metaphysics which remained was epistemology, which investigates these very limitations, unless one uses the term "metaphysics," as some still do, as synonymous with philosophy. As the terminology is hardly fixed, it is proper to say that in this book, for the most part, metaphysics is deemed to cover the field of speculation as to noumena, and as to things apparently beyond all race experience and possible powers of conception. There is reserved for later discussion the question whether a phenomenon, as distinguished from a noumenon, may be beyond experience and present conception, and still afford indirect evidence of existence.

CHAPTER II

THE UNIVERSE OF SCIENCE

EVEN the legitimate metaphysician is often a victim of a sense of universal illusion, and the more so in proportion as the relativity argument appears to him conclusive. The conviction that all phenomena are but appearances is apt to make him feel that nothing about him is real and as though he were living in a world of phantasmagoria. This has in some thinkers produced the extreme of solipsistic systems of philosophy, in which the ego is deemed the only reality and all else its dreams or modes of being.

A little reflection, however, must show that there is a reality underlying external appearances. In the first place, the phenomena occur independently of the will of the thinking ego. Inductively speaking, there must, therefore, be other forces. Again, there appear to be other individuals, other egos, the apparent exercise of whose wills causes mental reactions in the thinking ego, and who must, therefore, again speaking inductively, have an independent existence. Finally, there is a persistence in phenomena and in their relationships one with the other, which is highly probative of external reality.

Man finds certain phenomena repeatedly following others in a relationship to which his mentality reacts in the concept of cause and effect. The external experiences of the individual and the race repeat themselves with countless variations but with that persistence in type and sequence which alone renders life possible.

If you place your watch in your table drawer to-day, you know, or at least have a faith, practically equivalent to knowledge, that in the absence of comprehensible causes, it will be there to-morrow. A marble dropped from a window will fall to earth to-day and to-morrow. The great mountain chain that you see in the distance to-day will be there when you look from the same point to-morrow, and irrespective of your mood or will. In the last analysis man owes his life in the surrounding universe to a *faith* in these inductions of experience. What has been throughout fundamental human experience, man has an abiding faith will continue to be;

and so far as authenticated records show, this faith has not been misplaced.

Even, therefore, as man must live his life practically with this faith in the realities of the apparent, external universe subject persistently to uniformity and to law, so the scientist, even though with the mental reservation that, in themselves, he can never truly apprehend these realities, disregards this reservation and adopts as the basis of his work a faith in their external existence. He confines his investigations to phenomena and their relationships. Included in these investigations, of course, are his own cerebral and psychic phenomena, knowable in their appearance and relationships until the possibility of knowledge ceases as the inquiry approaches their inner nature and the examining ego itself. In other words, science assumes the common sense or vulgarian attitude with respect to the reality of external things; and the pragmatic sufficiency of its explanations and the accuracy of its predictions triumphantly justify this attitude.

Expounders of evolution are fond of illustrating the history of the race by that of the individual, which repeats the former. The race has had its childhood even as the man;

and even as the child finds a satisfying explanation of startling phenomena in fairy tales, so the infant race explained the phenomena of the external universe by crude notions of supernatural forces. When man, in the course of his development from the beast, first awakened to a sense of self-consciousness, began to reflect upon the surrounding world and noted its nice adjustments to his own wants, he was sure that some supernatural power had arranged things for his central comfort and well-being. True, he recognized serious maladjustments, such as disease and torment and death, but early man was an optimist. Generally with a vague hope of a compensation for these evils in a future life, of one thing he was at least certain, that the earth and man are the centre of all things. So it is assumed in the Biblical and in all ancient literatures, and to his simple logic, this proposition was unanswerable. The sun and the moon and the stars revolved about him. The sun was given him as his light by day, the moon and the stars, as his light by night. When he was hungry, there were plants and other animals to serve him for food. The waters in a perpetual cycle rose to the heavens to fall in life renewing power upon the fields, only to be carried by brook and stream and river, again to the thus exhaustless ocean. The love of male and female were specially designed to perpetuate his race.

Thirty centuries ago, it would have been a startling thought that the still existing maladjustments were the appalling evidences of nature's perpetual conflicts, of which what survived was the product best fitted to the environment, that what in nature was suitable to man's needs had not been created for him, but that he was the outcome of the contending natural forces that had created both; in other words, that man was not first created and then bountifully supplied, but was developed as the surviving product of the conditions, and that the things seeming to be created for him would naturally seem so to his budding, child-like intellect, since he was produced by, and never could have existed without them. From the fierce and perpetual battles of these contending forces, the race of man indeed emerged to the surprised face of nature, but could only exist and continue to develop and exist by ceaseless warfare against still existing hostile conditions, which only become less and less so, by almost imperceptible stages and in long ages of time, under the natural law of the survival of the fittest.

The development of scientific knowledge is like a great drama of many parts wherein, through long and conflicting stages of development, the mystic plot was step by step revealed, until, the obscuring mists gradually fading from before the observer's eyes, one part at least was ended and Nature appeared to him such as it is now conceived, an infinite in which countless universes exist and in which his own is but an atom. The ancient Greeks made surprising progress even in this direction, but their thought being interrupted, as we have already remarked, by the dark ages, it was not until several centuries ago in the times of Copernicus and Newton that the modern conception was reached. This conception is justified by the completeness of its explanations and by the fulfillment of predictions based on it.

Far from being the centre of all things, the earth is but an insignificant planet among a number revolving about the sun, and the sun is but one of scores of millions of similar suns or stars, each, in every probability, likewise surrounded by its attendant revolving worlds.

Our own, now balanced, solar system is doubtless the survival of innumerable collisions and catastrophes in æons and æons of past natural history. In the vast abysses of space, countless similar systems are in similar course of development, or of progressive disintegration to the nebulous chaos from which they sprang and from which the process will start anew.

The history of man has not sufficed to make changes of the stellar universe evident. The stars are situated at such inconceivable distances from us that changes in their relative positions due to their ceaseless motions could not be apparent to us in thousands of years. Light travels at the rate of approximately 186,000 miles in a second, and yet their distance is such that it takes the light from them thousands of years to reach us. A star we see to-day may have really ceased to exist thousands of years ago.

It has long since been demonstrated that the hordes of wandering meteors which the earth meets in its great journey about the sun and which it consumes in its atmosphere, and its sister planets, and the sun itself, are composed of materials uniform in relation to each other and to those which compose our own planet. Spectroscopic analysis shows also that even in the case of other systems and universes situated thousands and thousands of light years away, the materials are still uniform with our own. The very fact that the light of the distant stars reaches us through the medium supposed to fill all space, or at least all the space of the visible universe, again illustrates the significant fact that there is a oneness in all things and that, whether the philosophical attitude be one of so-called materialism or idealism, the universe, so far as we see or know it, is monistic.

The scientist may willingly concede space, as we know it, to be a form of thought, but that does not prevent him from regarding the form as interpretative of the potentialities of physical existence and motion. He looks upon it as the mental reaction to an external mode of being; and thus, in practical effect, he assumes for it the same reality as is assumed for it by the naïve mind. His universe is, or has been until Einstein, an aggregation of bodies inhabiting an infinite space.

Sir Isaac Newton's was the master intellect which discovered and formulated the laws of motion and gravitation, which until now have been considered successfully to account for all phenomena within their sphere. All prior conceptions were revolutionized when Newton's hypothetic laws that a body in motion will continue in motion forever in a straight line, unless interfered with by some force, and that bodies attract each other in proportion to the product of their masses and in inverse proportion to the squares of their distances, were found to be in accord with the actual motions of the solar system. The orbital motion of the earth, cast off from the sun in a gaseous and heated state, is the composite result of the centrifugal motion derived from the force of the sun's axial revolution when hurling it out into space and the pulling power of the gravitational attraction between the earth and the sun; and likewise with the motion of the moon, which is in the same way the child of the earth as the earth is of the sun.

It was Newton also who, in explanation of his own laws, had to account in some way for action at a distance. It is inconceivable that bodies at a distance can attract each other without some connecting medium. It is inconceivable that light can reach us from the sun or the stars without such a medium. Hence, the hypothetical ether, which, while it must be material in some sense, since it acts in some way as a material medium, nevertheless cannot be matter in any full sense of the word, inasmuch as, for one thing, it must be without weight; else it could not pervade all the vast abysses of space without special attraction for the great gravitational fields.

Science, which deals only with phenomena and therefore only with finite things, assumes always the law of cause and effect. In mortal experience every phenomenon is sooner or later found to have its cause. Omitting as immaterial to this discussion certain alleged achievements in the psychic field, there is no authentic record of a physical happening without a connected physical antecedent. One may will an event, but can only produce it with physical instruments acting on physical things. On such a basis, science has assumed that gravitation between masses, as well as the travel of light between them, must depend upon some physical medium.

It may be said, however, in passing, that the law of cause and effect is also a matter of induction, a matter of relative knowledge, without logical finality. It is based on human experience in the finite world. Certain phenomena are repeatedly observed to follow certain others and never to appear without them, from which is derived, by the principle of mental habit, the faith that the same order must in future continue. But it is also, perhaps, true to say that from the constant repetition of the experience of apparently related sequences of events, the human mind, from its nature, must derive the concept of necessary consequence, of cause and effect. In other words, the mind reacts to certain phenomena of antecedent and subsequent in the form of the principle of cause and effect, which is no more proof of its existence in the innermost nature of things, than is the reaction of the mind to certain motional phenomena in terms of space and time, proof that space and time exist as objective realities.

The great modern telescopes have revealed in the vast depths of space illustrations of universes in every stage of development called for by the modern hypothesis. Great nebular bodies, whether in the spherical or spiral form called for by the hypothesis, have been exposed, showing, in each case, the gravitational consolidation of enormous masses of matter in undeveloped or chaotic form, which, under the energy of the gravitational pull, develop heat and incandescence and motion until, in the course of their evolution, there is finally a consolidation into a great whirling, incandescent, gaseous sphere filling an orbit of a diameter of hundreds of millions of miles, just as our sun must have done until, in the course of its contraction, it threw off its first planet, the now far distant Neptune. Preciseness in such investigations is, of course, not to be expected, but the evidence is overwhelming that world systems are developed in some approximation to this fashion. As the incandescent bodies contract and cool in the space of hundreds of millions of years, planets like our own are developed, in some such way, which, in turn, are the mothers of a development of substance from simple to complex, from inorganic to organic, from unconscious to conscious, and from conscious to man.

Not only does this evolution take place in the formation of complex substances, but, as long since presaged, and as now rendered reasonably clear under the impetus of the recent discoveries of radium, beta rays, electrons and the like, in the development of the elements themselves. They appear to have been evolved from still simpler forms of matter, and finally from one underlying, universal substance or energy in different degrees or kinds of electric charge, which the unifying mind of science likes to think of in terms of the all-pervading ether.

In the same way that man has explored the vast reaches of space, he has explored the inconceivable littlenesses of the diminutive world. The ancient Greeks, by a pure process of deductive reasoning, had reached the conclusion that the divisibility of matter must cease when at last the elementary atom is reached. Like so many other of their deductions, this one was demonstrated in our own era, when the discovery of the chemical laws of combination of elements in definite and multiple proportions, followed by corroborating investigations, established that the material universe is composed of molecules and atoms, the molecules being the ultimate units of any given kind of complex matter, and the atoms being the ultimate units of the different kinds of elemental matter, which, combining with each other, unite to form the molecule. Recently the analysis has been carried even further, and it appears now probable that the atom itself is divisible into a central, positively charged nucleus about which revolve one or more negatively charged electrons. The atoms of different elements are supposed to differ from each other in some such simple way as in the number of the revolving electrons. In other words, the modern scientist is now demonstrating the probable truth of the dream of philosophers in all the ages, that all matter consists, in the last analysis, of one substance or energy in different conditions of electric charge or force.

It is, of course, only relative to speak of the world of molecules, atoms and electrons as diminutive. They are only so in comparison with human standards. The electrons revolve at a tremendous rate of speed about the central nucleus, and the spaces existing between them and the nucleus and between each other, are relatively equivalent to those separating the planets and their central sun, so that the little atom is in itself a universe. A drop of water may expose to the microscope a fully developed serpent; and these molecules, atoms and electrons, of which there are trillions in a single drop of water, are, to an imaginary being of the same measure relative to them as we are, relative to our universe, world systems in themselves.

In the light of the analysis of all substances into one substance, and that one substance a form of energy, we reach, by the methods of physical science, something similar to the philosophic deduction that all matter is a manifestation of a force in its nature unknown and unknowable to finite mind; and it must of necessity follow that the age-old controversy between the materialist and the idealist or spiritualist is, in its ancient form at least, a thing of the past. There can be no longer an opposition of a gross, inert matter to active spirit. Matter is reducible to force, or, if you please, spirit. The controversy still persists, however, in a restated form.

The materialist is the non-sensitive mind of the last chapter, who, having carried the analysis thus far, is content to say that the investigation is ended, that the universe rests on or consists of the manifestations of an unknowable, infinite and absolute monistic force which has developed all things up to the consciousness of man, that it is an arrogance of the human mind to conceive itself as of importance in the infinite whole, and that, with the disintegration of the human organism at death, disappears the conscious personality which for a brief instant of infinite time animated it.

The idealist, on the other hand, is lost in admiration at the power of that mind, which, hampered in its finite organism, can, nevertheless, penetrate so deeply into the nature of things, and feel so beautifully the inexpressible things beyond the reach of reason; and reverences it beyond all the tremendous physical forces that threaten it. Perhaps the idealist, in the last analysis, hopes to see demonstrated the fact that, even as underlying the appearances of the physical universe, there exists an unknowable force, so, underlying consciousness, exists an unknowable force which may constitute one with the first, so that this one Spirit underlying both consciousness and nature constitutes the universal noumenon sought by the philosopher.

CHAPTER III

MEASUREMENT AND POSITION

GEOMETRY is the most logical of all the sciences. It reaches the truth of its propositions by deductive reasoning, based, in the last analysis, upon so-called axioms. These axioms, such as the statement that a straight line is the shortest distance between two points, are truths which are admitted without question by all mankind. It used to be said by certain types of mind that such truths are inborn, but it is now generally held that they are inductions derived from universal individual and race experience. In other words, as is the case with all inductions, they are really taken on faith. Man has faith in the truth of that which has always been; he has faith in universal experience. He lives his life on the basis of this faith and it has never deceived him. Any exception to these universally believed axiomatic truths would be what is termed a miracle, and the burden is on him who alleges a miracle to prove it. At all events, in all the transactions of actual life and even in the more subtle investigations of science, at least until Einstein and his contemporaries, there has never been any demonstrated exception to the truths of axiomatic experience.

On the basis of these axioms as premises, the mind, purely as a matter of deductive reasoning and without any reference whatever to the external universe, except in so far as the external universe supplies the ideas which are the data of the mental operation, has proved the various propositions of geometry. The young philosophic student of that fascinating science, to those of uninquiring mind so dull, is lost in admiration at the external evidence of the truth or agreement with nature of these results of pure reasoning. When, on the basis of axioms apparently unrelated to the result, he finds that by the law of his mentality the sum of the angles of a triangle must always equal two right angles, and then proceeds successfully to test the accuracy of that proposition by actual compass measurement of triangles of every conceivable shape, it is no wonder that an ineradicable impression has been made upon his youthful mind of profound harmony in all things.

The result, however, is not so surprising when viewed in the aspect of the philosophic demonstration that space is but a form of thought. If extrinsic phenomena of motion and extension react upon the mind necessarily in terms of space, it is not surprising that the mind, given the persistence of these phenomena and their relations, our faith in which has never been disappointed, must, on applying the results of its logical processes to phenomena, always find them in harmony with itself. The mind is itself a part of nature and must find in it what it has taken from it.

It is, nevertheless, the fact, as Einstein at the outset emphasizes, that the truths of geometry, like the axioms on which they rest, are valid only so far as experience extends. This is the old thought of the non-Euclidean mathematicians who, during the last century, have amused themselves with the construction of self-consistent systems of geometry, different from the Euclidean geometry of this world, because based on different and imaginary axioms. We shall have more to say of this mathematics later.

Whatever one may think of the usefulness of this kind of speculation, it is well to remember that our accepted geometry is valid only so far as its axioms are true. It is a thought rich in possible consequences, that man's experiences which result for him in these axioms are only those general physical appearances which result from mass contacts. His mentality has been able to penetrate the secrets of the mass until it reaches some of the truths of its atomic and electronic structure. These cannot, however, be said to be matters of experience. The axioms are based on ordinary animal experiences, and it should not be overlooked, therefore, that it may be true that the same propositions which hold for the science of mass and ordinary mass velocities do not hold for the science of electrons and such high velocities as those of light and electrons, which are not experienced, but only inferred. What man has directly observed or experienced of motion consists of the motion of masses. When questions arise in relation to the motions and velocities of atoms, electrons and light, these being things beyond direct human experience, it cannot be validly asserted that the rules of nature in respect to them, however precisely one may ultimately learn them, are axiomatic. No one can assert positively that in the electronic world a straight line is the shortest distance between two points.

It has been asserted recently by a few scientists delving in these fields, that they have discovered such new phenomena as that electrons in motion gain in mass and lose in length. Such discoveries may be explained quite consistently with classic standards, but, as we shall see, Einstein prefers a more radical explanation. It is from this point of departure that his theory sets out, and it develops by discarding, for such phenomena, Euclidean and orthodox standards of space and time measurement and by the adoption of the ingenious systems of the non-Euclidean geometers, which he found ready for use. The classic thought is that space and time are relative concepts in relation to noumenal or absolute reality; but Einstein adds the thought that, even phenomenally, they are merely relative to our special condition, concealing a profounder physical truth.

Before we finally enter, however, into the analysis of his ideas of a special relativity, some explanation is needed of certain general conceptions of measurement and position in space, with which we shall conclude this chapter; and of the relativity of motional phenomena in the ordinary and ancient sense (not Einstein's), which shall be the subject of the next chapter.

Let the reader attempt as well as possible to conceive himself, as a mentality, existing in a space from which all masses have been removed. To ask the mentality to fix a location in this emptiness would be a manifest absurdity. Any point must of necessity be the centre of infinity, and, in the absence of physical boundaries or other material objects within the space, on which to establish points of reference, there can be no question of location. If now there be inserted into this space a uniformly moving ball, it would be true to state that to a disembodied mind, or, better, to a man traveling on the ball, and omitting from consideration its mysterious motion energy except in relation to change of position, the ball in motion would be the same as if at rest. The motion would be no more directly perceptible to the man partaking of it than is that of our own earth. It could not be said to travel unless there were other objects from which to measure the various changes of position, and then it would be impossible for our observer to determine whether the motion was in the ball or in those other objects.

If now we imagine a second and stationary ball in this infinite space and on it another observer, relative to him the first ball will move, while to the first observer the so-called stationary ball will appear to have the motion, and he will deem himself at rest. If the two balls now travel with uniform and equal velocity and in the same direction, they will be at rest with relation to each other and to the observers on each, but in motion in relation to a supposed stationary third ball, while to both observers the latter will be deemed in motion. In other words, whatever may be thought of the motion energy in moving objects, motion itself is purely relative and means to us nothing other than change of position. This subject will be treated more fully in the next chapter. We are here merely outlining a few basic matters necessary to the understanding of what follows.

Space is truly conceived only in relation to the extension of bodies and to their motion; and it is always so conceived, in the nature of our mentalities, at all events as at present constituted, in terms of three dimensions, length, breadth and thickness. No body can be said to hold a position except in so far as that position is determined in those dimensions with relation to some other body used as a body of reference. In attempting to describe, therefore, the position or motion of a body (succession of its positions), we must first designate the reference body in relation to which that position or motion is determined.

The fixing of the lapse of time between two events is, of course, only measurement in terms of the motion of some body of reference accepted as a standard. Thus, if it be said that someone has walked for the duration of a day, the meaning is that from the time his walk began until it terminated a point on the earth's surface has completed one rotation about its axis. The average velocity of the walk has reference to the distance covered during that rotation.

Distance is, of course, determined through physical measurement by fixed standards. We use measuring rods for that purpose which are, in the first instance, of arbitrary length, such as, for example, a yard stick. Now such a stick may, of course, be subject to relatively slight deviations in length due to recognized physical causes such as the wear of friction, changes in temperature, and the like. Allowing, however, for these well understood and relatively unimportant changes, it has always been assumed as axiomatic, on the basis of universal experience, that a measuring rod will always maintain the same length, and irrespective of whether it is in motion or at rest. This is on the same principle as that under which it has always been assumed, for example, that the desk on which one writes, in the absence of understood physical causes of removal or decay, will persist. We do not expect it to change in mass or to disappear while we are writing on it, and we have every confidence that on the next occasion when we shall want to use it, it will still "be there."

Since a stationary ether is supposed to pervade all space, the question naturally arises whether there is not motion of an object traveling in space, relative to the ether which surrounds it. Ether is, of course, invisible, but so is any gas, and yet it could not be said that a body would not have motion, relative to a surrounding gas and its particles. If the ether then has any of the attributes of matter, if, for example, it is made up of particles, and it were possible to identify any particle in any way, such, for example, as coloring it, the object would certainly have motion relative to that particle, in the same way as it would have such motion relative to any other body. The importance of this suggestion in relation to the Einstein theory will later develop in connection with the inquiry whether, if all known bodies are determined to be in relative motion, the ether does not constitute a standard of reference in actual rest from which their actual motions can be determined.

For the present, it is sufficient to state that we fix position and determine motion at any moment in relation to some standard reference system, which, for these purposes, is deemed at rest. Mathematicians use various kinds of reference systems, a few of which even those of our readers who possess only elementary mathematical knowledge will recognize when we designate them as those of polar co-ordinates or of co-ordinates of longitude and latitude or Cartesian co-ordinates. It will suffice here to explain briefly the latter system.

In determining the location of a point on a plane we adopt as our Cartesian reference system two lines or axes in the surface, intersecting at a right angle. The position of the point is determined by the lengths of two lines, known as co-ordinates, dropped from it to the respective axes and perpendicular to them. In determining the location of a point in space we use a Cartesian reference system which consists of three intersecting planes perpendicular to each other, such as the adjacent three sides of a box or hollow cube. To determine the location of the point, we ascertain the lengths of three lines dropped from it perpendicularly to the respective three planes, these lines being known as co-ordinates. Such a Cartesian co-ordinate system must, of course, be rigid and fixed to some body. If such a system be deemed to be affixed to one of the balls of our earlier illustration, moving or at rest, it is easy to locate, with reference to that system, the position of any outside point at any selected instant of time.

Of course, when we deal with moving bodies we need, besides our rigid Cartesian coordinate system, a method of fixing time and the system must, therefore, include, at each material point, clocks of identical construction. Classic mechanics not only assumes the permanence of our reference systems and of our measuring rods, whatever may be their states of motion, but also the absolute nature of time. That is to say, it is assumed that two or more clocks of identical and theoretically perfect construction will show, at any given moment, in the absence of physically comprehensible disturbance, the same lapse of time, and this, irrespective of their state of motion.

CHAPTER IV

THE RELATIVITY OF MOTION

SCIENCE has demonstrated that all known things are in motion. From mass to electron there is no rest. The atom and molecule are in ceaseless vibration, the electron moves about its nucleus, the earth about its axis and about the sun, the moon about the earth, the sun with all its attendant satellites in some direction that we do not know, and every distant star or sun, with tremendous velocities, in unknown orbs. These motions are, for the most part, only known to us inferentially. They are not matters of direct experience.

No living being ever suspected, as matter of experience, that the earth is rotating about its axis one thousand miles an hour and also is being hurled about the sun at a speed of over one thousand miles per minute. These facts have been demonstrated by logical processes, but certainly not by experience. The reason is that all objects on the earth move with it. The moving earth is, therefore, not moving in relation to an earth observer, but only in relation to objects outside itself, and, to the earth observer, it is these external objects that appear to move, and not the earth. It took countless centuries before men realized that it was the earth rotating on its axis, and not the sun moving around the earth, that caused the phenomena of night and day.

It is this ever present motion, as has already been pointed out, which impresses one, perhaps more strikingly than any other phenomenon, with the riddle of existence. Man has demonstrated scientifically that motion-producing energy as well as matter, which, in view of the philosophic analysis, one may perhaps be permitted to term static energy, is indestructible. The form of matter and energy may change, but their finite quantity in the finite world, once drawn from the exhaustless reservoir of nature, is invariable; and the energy manifest in matter in the form of motion, whether mass, atomic or electronic, is continuous and eternal.

The ancient Greeks understood the mystery of motion, if not on the same accurate scientific basis as we do, still, in its essentials. One of their old masters, Zeno, illustrated it in the famous problem of Achilles and the tortoise, Achilles typifying speed and the tortoise slowness in motion. We know, as a matter of experience, that in a race between the pursuing Achilles and the tortoise, both traveling in a straight line, he will overtake it, and that, as a matter of mathematics, given the distances apart at the outset of the race and their respective speeds, the precise time of capture can be figured. Nevertheless, the master undertook, as matter of pure logic, to demonstrate, in the following fashion, that the tortoise could never be overtaken.

To overtake it, Achilles must at some time reach the point at which the tortoise started, but, however slowly the latter has moved, it will in the meantime have advanced some distance beyond that point. When, however, Achilles has reached that point, and the tortoise a point in advance, even though the separating distance be considerably less than it was originally, the same problem is presented again, because from the tortoise's original starting point Achilles must again travel to the point to which the tortoise has advanced; but, however slowly the tortoise may move, it will again have advanced some little distance beyond the last named point, and so on again *ad infinitum*. Therefore, because, while

Achilles is traveling to each successive point to which the tortoise will have advanced, the latter must have proceeded some distance beyond, however infinitesimally small, Achilles can never overtake it.

The fallacy in the argument results, of course, from oversight of the truth that motion is continuous and indivisible. It is true, in a sense, that the distance which Achilles must travel in order to overtake his quarry is, mentally, infinitely divisible, and if the pursuer had to reach each of the infinite number of intervening mathematical points, his journey would have to consume an infinite time, which, of course, would render his object unattainable. But there is a difference between a distance infinitely divisible and an infinite distance, and neither is really conceivable or can, therefore, take part in any valid logical process. In the argument above set forth, the mind makes a pause at each of the supposed points from which the tortoise starts, evident in the statement that, when Achilles reaches such a point, the tortoise will have advanced some distance beyond to another point, which Achilles must again reach, but from which the tortoise will have again advanced while Achilles is reaching it. But, except in the mind of the person making the argument, no such pause occurs; once the race is on, neither Achilles nor the tortoise is at rest. The motion of each is continuous, indivisible, eternal.

The same profound truth may be illustrated in another fashion. As above noted, it has been demonstrated, by scientific means, that neither energy nor matter may be destroyed, but this truth is fairly self-evident in the nature of thought. It is possible, of course, to imagine changes in form and position, but impossible to conceive of the absolute destruction or non-being of a once existing physical entity. It may have taken men countless centuries to learn that the molecules of boiling water are merely separated in the wider distances of invisible steam, or that the burning of a log is a mere union of its atoms with atoms of surrounding substances in invisible form, but it was always an impossible mental feat to conceive of the absolute nonentity of any substance or force once existing, however puzzled the observer might be to account for the *apparent* disappearance.

When a fast moving railroad train comes to rest, we know, scientifically, that its tremendous energy has not been destroyed, but has passed on into various forms of non-mass motion such as heat. Nevertheless, it is impossible for us to conceive how the transfer is made, and difficult, if not impossible, as some believe, to conceive of the train coming to rest without the attendant conception that between its original motion and the point of rest it must pass through every intermediate velocity. Although the velocities become smaller and smaller, each but a fraction of the last, still, as there will always be some velocity, even though the most infinitesimal fraction of the preceding one, how can it be possible that there shall be ultimate rest? However one may answer such questions, it is reasonably clear that we are here dealing with manifestations of an eternal energy, in itself inscrutable, made apparent in motion, as it translates itself from mass to atom, from atom to electron, from electron to ether.

Such considerations are not, strictly speaking, essential to an understanding of the Einstein theory any more than are various interesting illustrations of the relativity of motion, of a character utilized by Einstein himself. But the mystery of relative motion is his basic problem, and all such considerations and illustrations are useful in preparing us for the very remarkable and revolutionary ideas of nature which are the result of his theory. They dispose the mind to perceive more clearly than it would naturally do, without reflection on such phenomena, that its natural manner of apprehending the universe is the product solely of experience, which, of course, as these considerations illustrate, does not cover the field. Man only knows things as his senses apprehend them or as he may infer them in terms of sense experience. The subtle vibrations of air and ether become for him harmonies of sound and sight. But there must be, in nature, infinite varieties of inexpressible harmonies, in degree and kind, beyond the reach of his poor tools. Such considerations, then, by shaking our confidence in every day points of view, predispose us for the reception of revolutionary truths so subtle that they have escaped ordinary experience.

Let us take a few fanciful illustrations in order to further this mental attitude. Light travels at the rate of 186,000 miles a second. Let us imagine that by some miracle a human observer is transferred instantaneously to a star so distant from the earth that it takes light, though traveling at that tremendous velocity, 109 years to travel from the earth to that star. Let us suppose our observer to be gifted with supernaturally powerful eyes which he directs toward the earth. He would, of course, not see the things then happening here, but those that have happened 109 years ago. If his eyes should be directed to that part of the earth represented by the battlefield of Waterloo he would see the fierce charges of Napoleon's old guard upon the battalions of Wellington. If our observer were now to travel from his star farther away from the earth at a velocity exceeding that of light, and with his eyes constantly directed toward the earth, he would see, as long as he traveled thus, the panorama of human history, from the point of the battle of Waterloo, unfolding itself reversely through all the stages of precedent events, because as he travels away from the earth faster than the light, he would at each stage of his journey be overtaking the light reflected from earlier and earlier events.

In the same way, if a person were to travel on this earth at the rate of thirteen miles a minute, which is approximately the speed of sound, and a gun were to be fired at the point and instant of his outset, the sound would remain in his ears as an accompanying roar, because the air vibrations, traveling precisely at his speed, would naturally keep him continuous company, allowing, of course, for their gradual destruction through interfering forces. If, however, this traveler's velocity were to exceed thirteen miles a minute, the sound of an explosion taking place a moment after his departure, however powerful, would never reach him, and for him that sound would not exist.

Taking again our observer with the supernatural eyes and placing him upon the earth and letting a self-luminous clock, its hands pointed at twelve o'clock, move away from him, with the velocity of light, with its face ever toward him, it is easy to see that the hands, for the observer, would always show a lapse of time, after twelve o'clock, one-half of that shown on a clock of identical construction at his side, because at any moment of his observation, for example at five minutes past twelve on his own clock, he would not be observing by the light which left the traveling clock at that moment, but by that which had left it two and one-half minutes before. This is in accordance with the orthodox idea of relativity. It may be remarked, in passing, that according to the special theory of relativity, as we shall see, the hands of the moving clock would never show an advance from twelve o'clock.

We shall now again, but in a little more detail, illustrate the relativity of motion along those lines which are directly connected with the Einstein point of view. We allude to the impossibility of detecting one's own uniform motion except in relation to another body, and to the impossibility of allotting such motion to one rather than the other. This has been mentioned by the early Greeks and must date back to men's earliest experiences, but perhaps we have not thought about it sufficiently. It is well illustrated by such a fact as was mentioned at the beginning of this chapter, man's ignorance, by direct experience, of the motions of the earth on its axis and about the sun.

If a man travel on a train moving with uniform velocity, imperfect as the uniformity must be under human conditions, we know, nevertheless, that he is frequently deceived as to his motion relative to that of objects which he passes. Often he realizes that, without his prior knowledge of his own motion, he would be unable to determine that it is not the surrounding landscape which is passing him at rest. Under such conditions it is often impossible to determine whether trains which he passes going in the same direction are at rest or also moving, or whether he himself be not at rest and the observed train moving, and in a direction the same or the opposite of its actual direction, according to the relative velocities.

In space, to an observer on the only body therein, moving with uniform velocity, there being no body of reference, there would, of course, be no motion apparent. Suppose now the observer himself at rest on and relative to a platform moving through space at uniform velocity, and at his side, within reachable distance, a similar platform, level with his, traveling in the same direction and at the same velocity. This velocity might be as tremendous as you care to imagine, and yet to the observer both platforms are at rest. The observer might throw a plank from one platform to the other and cross it or shake hands across the intervening chasm with a similar observer upon the other platform, with perfect safety. If, however, one of these two platforms were at rest and the other in motion, or if both were moving but at different velocities in the same or in different directions, to an observer on either it would appear that his own platform was at rest and the other in motion. To such an observer, even if the other were to collide with his platform, whether that phenomenon, as viewed by an outsider, were due to the motion of both or of either, the appearance would be that he was at rest and shocked by collision with a body moving toward him.

A man may be walking rapidly west, but to an observer on an imaginary planet, accompanying the earth on its orbital journey, but without other motion of its own, the man's velocity west being so small relative to that of his motion about the earth on its axis in the opposite direction, his travel would be clearly east and at a much greater velocity. To an observer on the sun, who would observe the effect both of the earth's axial and orbital travel, the man's motion would be of an enormously different appearance. To an observer on a body of reference entirely outside the solar system, it would probably be apparent that earth and sun together are traveling at an even greater velocity in some direction now unknown by men, so that to such an observer the walking man's other motions would be lost in the complex.

Einstein, in illustrating these truths of the relativity of motion, is fond of assuming a train traveling in a straight line at uniform velocity along an embankment assumed to be at rest. For the purposes of the many illustrations that he makes on this assumption, it is well to imagine the train of enormous length, and, of course, to assume the air above the embankment entirely removed.

If, under such conditions, an observer on the train were to drop from the car window a stone, it would, of course, have the velocity of the train in the direction of the train's motion, and would, therefore, in its journey to the earth, to this observer, the train being his body of reference or co-ordinate system, travel in a vertical line, that is, a line perpendicular to the floor of the car. To an observer on the embankment, however, using it as his system, the stone would appear to travel in a parabolic curve. Which is true? Has the stone traveled in a vertical line or in a parabolic curve? Each is true, dependent upon the reference system selected; motion is relative.

Let us again suppose this train traveling at the uniform velocity of one mile a minute and on it a man who walks from its rear towards its forward end at the rate of a mile in fifteen minutes. What is the velocity at which this man is traveling? To an observer on the train, using it as reference system, he is traveling at the rate of four miles an hour, but to an observer from the embankment system he will be traveling at the rate of one and one-fifteenth miles per minute, since, when the rear end of the train has reached in one minute the distance of one mile from the point at which it was when the man began his walk, the latter will have walked forward an additional one-fifteenth of a mile. Of course, in making their computations the observers ought to allow for the fact that the observations are made by means of light reflecting the man to their eyes and that this takes varying times, however small, according to the distances at the various moments of observation. The velocity of light, however, is so enormous compared with the other velocities involved in this problem or any other of ordinary experience, that this factor is too trifling to take into consideration. Theoretically, however, after allowance for it, we should expect the ascertained velocity of the walking traveler to conform to the result above indicated. We shall later see that the Einstein theory denies this result.

Suppose our observer on the train to be computing the direction and velocity of a flying bird. It is manifest, always assuming that the observer knows nothing of his own motion, that if the bird flies parallel to and in the train's direction and at the same speed, it will appear to him at rest. If the bird, as judged from the embankment, flies parallel to the train's motion of a mile a minute, but in the opposite direction at the same velocity, beginning its flight from a point beyond the front of the train, it seems, as judged from the train, to be flying towards, past and away from it at the velocity of two miles a minute. If the bird travels along the embankment parallel to and in the direction of the train's motion at a velocity of two miles a minute, to the observer on the train, it appears to be traveling away from him at a velocity of one mile a minute. If the bird travels in a straight line perpendicular to the embankment, to the observer on the rear platform of the train, traveling away from the point at which the bird's perpendicular journey begins, it seems to be traveling away from the rear end of the train in a straight line not perpendicular to the floor of the train, but on a slant. In all the supposed cases, while the motion of the bird appears to the train observer at different velocities and sometimes in different directions than to the embankment observer, to both it always appears to be either at rest or in uniform motion in a straight line.

From the foregoing, it is clear that the laws of nature, with respect to mass motion at least, are the same whatever reference system be selected, so long as the universe be one of reference bodies either relatively at rest or traveling relatively at uniform velocities in straight lines; that is to say that, whatever system be used, in such a universe the laws of Euclidean geometry and classic mechanics prevail. To the observer on one reference body an object might seem to have a velocity or direction different from that apparent to an observer on another, but travel would always appear uniform and in straight lines and, given the relative motions and velocities, capable of mathematical transformation from the values of one reference system to those of another. Any change in the object's motion, during the period of observation, since the relative velocity of the systems does not enter into the change, ought, therefore, to be expressible by the same mathematical formulation on any system. The problem at this stage of the inquiry is not complicated by the consideration of accelerated, that is, of rotatory or otherwise non-uniform motion of either system.

The reader ought to consider, however, that there is no actual experience in this world of the uniform or non-accelerated motion in straight lines described in Newton's law of inertia, to the effect that, unless the condition of rest or motion is changed, that is, accelerated, by interfering forces, a body at rest will remain at rest forever, or, if acted on by a force or forces, will move forever in a straight line in the direction of the applied force or in that of the resultant of the applied forces. Our universe being the scene of ever present conflicting forces, this law is theoretic only. It is based on no actual experience, but splendidly confirmed by the accord with facts of mathematical calculations based on it after due allowance for the known magnitudes and directions of the interfering forces.

Thus, while we have not in the known universe any case of perpetual motion in a straight line, while the planets move in elliptical orbits, their orbits have been precisely calculated on the assumption that they would move in straight lines forever were their centrifugal impetus not forced into the ellipsis by the known centripetal pull of the interfering body. Astronomical and other mechanical laws have, until the present time, been demonstrated, even though the motions be non-uniform and in curved orbits, on the basis of Euclidean geometry and Newtonian mechanics. Nevertheless it must be borne in mind that this geometry and mechanics may be approximations only, true for the conditions of ordinary experience, but needing a more subtle extension, if not reconstruction, for velocities and attendant phenomena beyond ordinary experience, such as those of light, electricity and atomic structure.

The principle, just explained, of the equivalence of all reference systems for the mechanical laws of nature may be plausibly extended to cover all the laws of nature and as thus extended may be summarized to mean, following Einstein's language, that natural laws run their course with respect to one uniformly and rectilinearly moving co-ordinate system in the same way as with respect to another similarly moving co-ordinate system. This statement is called by Einstein *the principle of relativity in the restricted sense.*

The writer assumes him to use this expression, not in the new sense of his special theory, but in the sense of classic science in which relativity has been understood for hundreds of years, limited, however, to phenomena of uniform, straight motions. The reason for that limitation is only that the consideration of accelerated motions is deferred because of the greater complexity of the problem of those motions. We shall proceed for the present on this assumption of the meaning of the restricted principle, but for important reasons which will appear later shall re-examine this point of view in Chapter VII.

Einstein's first theory of relativity which we are now about to describe, he calls the "special" theory as distinguished from the "restricted" principle of relativity. The word "relativity" is used by him, therefore, in three senses: first, the restricted sense, just explained, second, the special sense, about to be explained, and also applicable only to a universe of uniform, straight motions; and, finally, in the sense of his extended or "general" theory of relativity by which he undertakes to extend the results of the special theory to a universe such as that in which we live, containing rotatory and accelerated motions.

CHAPTER V

THE SPECIAL THEORY OF RELATIVITY

It is apparent from the considerations set forth in the last chapter that the principle of relativity, in the restricted sense, means that, whatever may be the apparent direction or velocity of one body of reference with respect to another, the particulars, i.e., the co-ordinates, direction and velocity of a moving object outside of both, may be readily transformed or calculated at any instant with respect to either by observers on both, if they know the relative motion and the position of the two reference bodies and those particulars relative to their own. For example, although the velocity of the bird of the last chapter, traveling with reference to the embankment at a speed of one hundred miles per hour, appears, to the observer on the train traveling in the same direction at sixty miles per hour, to be forty miles per hour, given to this observer the fact of his own velocity and direction relative to the embankment, or what is to him the same thing, the embankment's velocity and direction relative to the train, he can readily calculate the speed of the bird with reference to the embankment at one hundred miles per hour. The force necessary to increase the speed a certain amount will be calculated at the same quantity by both observers, and by means of equations in which the relative velocity of train and embankment will cancel out.

In the same way, the observer on the rear platform of the train, noting the bird's travel away from the train at a slant, given the direction and velocity of his own motion on the train, will be able to calculate the velocity and perpendicular direction of the bird's flight with respect to the embankment. The problem of the velocity of the man of the last chapter walking in the train can be directly figured, even with the complex allowances called for by the finite velocity of light, by taking into consideration the velocity of the man's motion relative to the train, the train's velocity relative to the embankment, and the velocity of light.

These things being true, the ordinary reader of Einstein's treatise on Relativity is at first thrown into confusion by the fact that the

Professor, after leading him thus far, states that, if a beam of light take the place of the bird in traveling along the embankment, it would be a contradiction of the restricted principle, if the observer on the train found that the apparent velocity of the beam traveling in the same direction as the train would be less, or, if traveling in the opposite direction, greater than its velocity as figured from the embankment. He himself has already told us that, according to the classic or restricted principle, the bird's apparent velocity from the train is the bird's embankment velocity less or plus the train's embankment velocity, according to whether the bird is flying with or against the train's motion; also that the velocity of the man walking in the train is, to a train observer, the man's embankment velocity less the train velocity; and that the restricted principle is not violated by a difference in velocity, so long as the motion is still straight and uniform. But for the train observer to find that the ray of light will appear, from the moving train, to have a different velocity than 186,000 miles per second, although it is known to move from the embankment at that velocity, is now stated to be a contradiction of the restricted principle.

This confusion is due to an obscurity of exposition in the treatise. In harmony with what we have said at the end of Chapter IV, we are assuming for the present that what the Professor means his readers to understand here, although he has not yet mentioned the subject, is that certain conclusions, to be examined hereafter, drawn by him from the Michelson experiments, first performed in 1887, have established the law of *a constancy in the velocity of light at 186,000 miles per second irrespective of the motion of the observer;* that, assuming the truth of that law, if the natural calculation or transformation under the restricted principle produces a different result, that principle cannot be true; and, therefore, that either the restricted principle must be rejected or a harmonizing new truth must be added to it, which will render true both the restricted principle and the fact of the constancy of the velocity of light. This harmonizing new truth and the surprising results that flow therefrom, the Professor offers in his special theory.

In other words, the Professor assumes that it is an established law that, no matter what the velocity or direction of the train of his illustration, the light, unlike the bird, will always have, to the train observer, or to any other for that matter, the same velocity. This supposed law of nature will not, therefore, be in harmony with the restricted principle, since the mathematical transformations, based on the differing velocities of the systems, ought, on classic grounds, to yield differing and not constant results for the apparent velocity of light.

Another construction of the Professor's meaning will be examined in Chapter VII. It is, briefly, that he may mean to advance the proposition that the constancy of the velocity of light in the sense just explained ought logically to follow, even if the Michelson experiments had never been made, from two cardinal principles, the relativity of motion and the perfectly comprehensible fact that light has a constant velocity of 186,000 miles per second irrespective of the mass motion of the body which, by its atomic motion, emits it. This proposition, as will later appear, does not seem sound to the writer. In this book the expression "constancy of the velocity of light" is used in the sense derived from the Michelson experiments, to wit, a constancy irrespective of the motion of the observer, except where it is clearly stated to refer to the primary sense of a constant velocity irrespective of the velocity of the light-emitting body.

Whichever be the correct construction of the Professor's logical method, however, it may be remarked here, anticipating somewhat, that the final reconciliation effected by the special theory lies in the application of a method of transformation which differs from the classic one by rejecting the constancy of time and distance measurements in the various systems. That is to say, if it be true that the differing motions of two or more observers have no effect upon the apparent velocity of light, the explanation must be that their relative motions must have some remarkable effect upon their standards of distance or time measurement, or both. The time units or length units, or both, of all the observers must be different, and in a surprisingly compensatory way, in order always to produce the same velocity.

Therefore, before proceeding further with the explanation of the special theory, we shall follow Einstein's example, who, in order to prepare us for acceptance of the wondrous results of the Michelson experiments and of the special theory of relativity which is their product, diverges into an analysis of time and time measurement, intended to suggest, if not to prove, that time itself differs according to the state of motion of the reference body on which it is figured; that clocks of identical construction will register differently according to their relative states of motion. In order to compare the time of clocks on different systems, or even at different places on the same system, we must start by synchronizing them, by establishing a moment of simultaneity.

We are asked what we mean by a simultaneity of events. If a flash of light is emitted at each of two separate points on our railroad embankment, we are confronted with the problem of determining whether or not the flashes are simultaneous. To determine this, let us imagine that there be placed at the two points in question clocks of identical construction and at a point along the track, precisely midway between the two points first mentioned, an observer with a similar clock and with an arrangement of mirrors so contrived that the observer can at the same time see reflected both of the first mentioned points. On the stipulation or assumption that the light from the two flashes will travel along the embankment from both these places the equal distance to the position of the observer in equal periods of time, we are told that the two flashes are said to be simultaneous if the observer sees them in his mirrors at the same instant, that is, if the flashes become one mental image. If the identical time at which the flashes are set off at the two points in question be known, the time on the clock located at the position of the observer will show the velocity of the light transmission, assuming the clocks to have been properly synchronized; and conversely, if we have not synchronized the three clocks, such an experiment should enable us to do so.

If now, however, the observer, instead of being placed at a stationary midway point, be placed upon a moving train in a position that, at the instant that the flashes are emitted, is coincident with the midway point upon the embankment, and he be similarly equipped with apparatus, it is apparent that his motion toward one of the flashes and away from the other will result in the fact that he will see the flash toward which he is traveling a short time before the flash from which he is traveling, so that to him the flashes will not be simultaneous.

These considerations are perfectly obvious from and consistent with the classic principle of relativity, for in each of the cases supposed the simultaneity of the flashes can be deduced; even by the observer on the train, given to him the knowledge of his own motion and velocity. Nevertheless, Professor Einstein remarks, after the foregoing explanation of the relativity of simultaneity, that:

> "before the advent of the theory of relativity, it has always tacitly been assumed in physics that the statement of time had an absolute significance, *i.e.,* that it is independent of the state of motion of the body of reference."

That is true, but not as applied to such problems as that of the train and the light flashes. Physicists, like all other thinkers, recognize that the relative state of motion of the observer must be allowed for in such computations; and there is, therefore, in such cases, no such "tacit assumption," except in the sense that for most earthly observations, even the most delicate, the speed of light is relatively so enormous, that the very minute allowances required to be made by reason thereof are negligible.

But Professor Einstein goes on to say, in reference to the man walking within the train, that the conclusion that he traverses the same distance in the train with respect to the embankment as with respect to the train, in each second of time, is shown to be fallacious by the foregoing considerations, inasmuch as

> "the time required by a particular occurrence with respect to the carriage must not be considered equal to the duration of the same occurrence as judged from the embankment (as reference body). Hence it cannot be contended that the man in walking travels the distance w relative to the railway line in a time which is equal to one second as judged from the embankment."

It is true that the relativity of simultaneity suggests to the mind the possibility of a different measure of time on one body of reference from that obtaining on another, the possibility that time measurement *may* be influenced by relative motion; but the suggestion is, logically, in no way connected with the above described phenomena. They merely present, in a more subtle form, the same considerations that we have had before us in illustrating relativity with reference to mass motions.

In other words, the only logical consequence of the relativity of simultaneity, classically speaking, is that, in making calculations of the simultaneity or non-simultaneity of events, we must remember to take into consideration in the orthodox way the motion of the observer in relation to the light or sound acting for him as messengers of the events. Here again there is some obscurity in the Professor's exposition. He has not yet presented the revolutionizing views that become later apparent. It is, therefore, again assumed, for the present, that what he means is merely to illustrate from these relativities and consequent mental confusions, the *possibility* of those assumed actual differences in time units, which, we shall presently see, figure in the special theory as part of the explanation of the assumed fact of the constancy of the velocity of light.

Reflection, in the light of what follows, indicates, however, that his real meaning goes much deeper than he has, up to this point, stated or hinted; that he means, as explained in Chapter VII, to derive a denial of absolute simultaneity on different systems from the relativity of motion and from the constancy of the velocity of light in the primary sense. It is sufficient here to indicate that this meaning is that, because motion is relative, because an observer cannot detect his own motion except in relation to another body, motion is not actual, that is, that there is no motion in relation to the ether or other supposed absolutely stationary medium of the travel of light, that the train, therefore, is not actually moving any more than the embankment, and that it follows that there can be no actual simultaneity of two events, one on each of two relatively moving systems, which have by the fact of relative motion, different *kinds* of time. This thought will rendered clearer later on.

In the meantime, we cannot leave this subject without an attempt at further clarification, necessary even at this point. We must be careful, though, not to introduce prematurely any Einsteinian, as distinguished from classic, principles. The Professor has not, up to this point, stated, or at least not clearly, that the prime difficulty in establishing the simultaneity or non-simultaneity of his light flashes arises from the fact that *neither the train nor the embankment observer has any basis for considering his own system at rest.* Let the reader divorce from his mind the age-old, naïve assumption of earth stability, and conceive of these two systems out in space. It is impossible for an observer on either system, or at any other known place, for that matter, to determine whether either is at rest or that both are not in motion.

If we were permitted to imagine an observer at absolute rest in space or in the ether, the light-carrying medium, to such a mythical observer, by the simplest calculations of the actual motions and distances perceived, the absolute simultaneity or non-simultaneity of the flashes of Einstein's illustration must appear evident. In fact, in that illustration the observer on the embankment seems to be assumed to be in actual rest and the train in actual motion. It is true, however, that he cannot be certain of this. The observer on the train, likewise, has no means of determining whether it is his train which is moving or the embankment, but deems himself at rest. Nevertheless, if on either of these systems the observer measures off 186,000 miles on either side of his central observation point he is, on classic grounds, on the assumption that he is at rest, entitled to be satisfied, knowing the velocity of light to be 186,000 miles per second, that the flash from each of his two measured limits will reach him at the observation point in one second of time.

On the other hand, in the absence of such assumption of rest, if the flashes do so reach him together, it may very well be that they were simultaneously emitted one second earlier and that his system is at rest in the ether;

or it may just as well be true that his system is not at rest and that the flashes were not simultaneous. The light traveling from the flashes wherever originally emitted does not partake of the motion of either system, if there be any such motion. Its velocity is constant in space or in the ether, independent of that of its source. If each observer insists that his own system is at rest and the other in motion, there is no reason for either, upon receiving simultaneously the two flashes, to doubt that they were simultaneous in origin.

But, as we have seen in Einstein's illustration, if received together by either observer, they cannot be so received by the other, if the latter's system be moving relatively to the first. Classically speaking, in that event, the relative motion must be allowed for in the calculation. Absolute simultaneity could be determined only from a system in absolute rest. In case of any motion, then, actual or relative, how can it be possible to synchronize the clocks on the two systems, or even on one of them, if in motion, since in the last analysis such synchronization must be by signal through the hypothetic ether? Let us suppose that at the instant the midpoints of the two systems are coincident the clock at that point on each system is set at 12 o'clock, and a flash then and there set off, its beams traveling in all directions, each observer having previously instructed his assistants to set the two clocks, 186,000 miles away on either side, at one second after 12, the moment each assistant receives that flash on his clock from the midpoint. Now, to take the simplest case, if we assume one of the systems to be at rest, it is apparent that, while both distant clocks on that system will thus be set at one second past 12, the forward clock on the other system, since it is moving forward, will not be overtaken by the pursuing beam until later, but will still be set, on such receipt, at one second past 12, while the rear clock, traveling towards its beam, will be similarly set, though receiving the flash much earlier. The observer on this system, however, deems himself at rest and the other system in motion in the opposite direction. Therefore, for him the forward clock on the system now appearing to move, which corresponds to the rear clock on his own system in the case first supposed, will similarly be set at one second after 12, at a time when on the relatively stationary system more than one second will have elapsed, and so on. This renders it evident merely that, because under the circumstances supposed it is impossible to synchronize the clocks on two relatively moving systems, except by the intervention of an as yet undiscovered observer in absolute rest, it will be impossible to *determine* actual simultaneity. This is far from proving that there is no such thing.

The confusion and difficulty of this subject arises from the indubitable fact that we are unable to determine the motion of two systems, except relatively. It is only our theoretic absolutely stationary observer who could determine whether one was in actual rest and the other in actual motion or whether the appearance from each was a composite of actual motions in each. The subtle thought behind these strange results will be further discussed in Chapters VII and VIII, where we shall attempt to explain Einstein's idea that there is no such thing as actual rest or actual motion; not only that motion of a body cannot be detected except in relation to some other body, but that it cannot be detected and does not exist in relation to space or the stationary ether classically supposed to fill space.

The material point at present is that by the discussion of the relativity of simultaneity, and the mention of the possibility of the relativity of distance as between different systems, Einstein, whatever may be the appositeness of his illustrations, is suggesting that the remarkable result, assumed to be proved by the Michelson experiments, that light not only always travels at the same velocity but will always appear so, whatever the velocity of the observer, may be due to the fact that both time and distance units, seconds and yard sticks, are subject to actual increase or decrease, according to circumstances. The proposition is that to an observer at rest light's velocity is 186,000 miles a second, but that if he move towards the approaching light at the velocity of 100,000 miles a second, the velocity of light will not appear to him 286,000 miles per second, but still 186,000 miles, because both his second and mile stick are correspondingly different.

The special theory and all its marvels rest on this thought. Of course, results correctly deduced on mathematical lines from a given premise must be expected to be startlingly contradictory of all previous notions of nature, if that premise is a supposed fact of nature contrary to all previous experience. If it could be demonstrated that a man existed with the power of accomplishing instantaneously anything he willed, it would not be startling, this premise once admitted, to find the universe becoming a wonderland in which everything would appear to us wandering children in a topsy-turvy condition. A stone might be willed to stay in a suspended condition, half way down its fall from the top of a precipice; great cities might be created with the speed of thought; the earth might be stopped in its perpetual journey, without falling into the sun. This digression is intended merely to emphasize the fact that all the startling results which Professor Einstein develops have as their major premise this cardinal assumption with respect to the constant velocity of light.

To put it clearly, at the risk of repetition, that assumption is that the velocity of a ray of light, measured from any reference body, at rest or moving in any direction and with any velocity, will appear always 186,000 miles a second. If the earth were stationary, a ray of light from the sun would reach it at the rate of 186,000 miles a second in approximately eight minutes. If the earth now were to travel directly into the sun at a velocity of 100,000 miles per second, by the classic principle of relativity, to an observer on the earth not aware of its motion, the velocity of the ray should appear to be 286,000 miles per second, reducible to 186,000 miles per second if the calculator knew of, and introduced into the calculation, his own motion; but, according to Professor Einstein, the ray's velocity, in spite of this motion of the earth, will still appear to be 186,000 miles per second. In the same way, his belief is that if the earth were traveling directly away from the sun at the speed of 100,000 miles per second, the apparent velocity of light traveling from the sun would not be 86,000 miles per second reducible to 186,000 miles per second when the observer took into the calculation his own motion, but would still be 186,000 miles per second.

The Michelson experiments upon which these results are predicated were based on the prevailing conception that the earth is moving through space relatively to a supposed ether, stationary as to it, and, in fact, as to all masses. The intervals between the ultimate particles of masses are supposed to be filled with the ether, which is undisturbed by the mass motion. The ether, furthermore, is assumed to be the medium of the transmission of light. These things being true, the velocity of light as computed on the moving earth ought to appear different accordingly as it is measured moving in the direction of the earth's motion or perpendicular to that direction. Regarding the earth as stationary and the ether moving by it in the opposite direction to that of the earth's actual motion relative to the ether, which, for present purposes, is the same thing as the actual fact, there should be an ether drift, and the velocity of light in the ether should be affected, as judged from the earth, in precisely the same way as the velocity of a swimmer would be different, as judged from the bank, accordingly as his direction was against or across the current. The light, like the swimmer, ought to travel across the current and back faster than against it and back, and of course much faster with the current than against it.

It was hoped that by properly devised methods involving sufficiently delicate instruments this ether drift could be detected. The Michelson experiments have been made in relation to the journey across and back and that against the current and back, and with entirely negative results. No difference in velocity could be detected, so that Professor Einstein, being satisfied with the accuracy of the tests, accepts the immediate conclusion that the apparent velocity of light is the same whether the light travel against or across the direction of the earth's motion and back again, and enlarges it to the more remote conclusion, by no means established even if the result of the experiment be accepted, that this apparent velocity will be the same in whatever direction the beam travel relative to the earth's motion, and whether directly with or directly against the supposed ether drift. The remote conclusion is an hypothesis, dependent, like all others, on empiric proof. The final decision as to the sufficiency of the experiment and the truth of the immediate conclusion must, of course, be left with specialists. It is hardly presumptuous, however, to point out that the results may either be not accurately determined or susceptible of other explanation.

In the first place, the expected detection of the drift assumes an ether stationary relative to the earth. Similar experiments as to the velocity of sound would, of course, show negative results, because the air, which is the medium of sound transmission, partakes (disregarding winds) of the earth's motion. The air does not move relatively to the earth. In our present state of knowledge, it cannot be assumed, however, that a large block of ether surrounding the earth to a considerable distance, or at least a smaller part of it in which the experiments are of necessity made near the surface of the earth, does not also partake of the earth's motion through adherence or some other cause. This is not an improbable hypothesis and its truth would be far from as revolutionizing to accepted views as the supposed law of the constancy of the velocity of light.

In the second place, the experiments, while conceded to have been conducted with apparatus of marvelous delicacy, nevertheless depend upon measurements so minute that error would not be surprising. The velocity of the earth in its journey about the sun and relative to the ether is some 18 miles per second. The velocity of a ray of light being about 186,000 miles per second, its velocity traveling in the direction of the earth's motion or against the ether current should turn out to be 185,982 miles per second, and in the opposite direction or with the current 186,018 miles per second. It was not possible, of course, to measure directly these velocities, nor, even if possible, would it be likely that a difference so minute could be detected. The scheme of the experiments was to detect not velocity magnitudes, but velocity differences, that is, to detect a difference in time in the return of light rays sent to and fro for equal distances in different directions with reference to the current.

The actual scheme was to send two portions of a divided beam of light in each case for the same aggregate distance, one portion to and fro between two mirrors with and against the ether current, and the other portion to and fro between two mirrors across the current, with the idea of determining whether the return of the two portions of the ray to the common starting point would be at the same or at a different instant. The apparatus used was revolved so that all directions might be utilized, the needed ones not missed, and possible defects of measurement in the apparatus eliminated. The calculation on orthodox standards was that, under the conditions of the experiment, the difference in time between the two returns should be a fraction of a billionth of a second. In view of that minute quantity, it is not too much to say that the result may not be final, even though the interference apparatus is pronounced by scientists to be capable of detecting it; and especially as further experiments have been made and at high altitudes where the possibility of the ether partaking of the earth's motion through adherence has been reduced, which, while not conclusive, have to some scientific minds indicated the possible detection of a drift. Further experiments of a similar character are about to be made.

Furthermore, Lorentz advanced a physical theory strictly in accord with orthodox ideas, and by no means intrinsically impossible or improbable, which would account for the failure to detect the drift. This is that bodies contract in the direction of their motion through the ether. If it were true that the arms of the Michelson apparatus in which the light beam was transmitted were shortened while pointed in the direction of the earth's motion and retained their original length while pointed across that direction, the distances traveled by the ray might still be the same in spite of the earth's motion; or if not quite the same, that is to say, if the shortening process did not amount mathematically to a precise compensation, it might still so largely reduce the expected trifling difference of time in the journey of the ray in the two different directions, as to render still more improbable its detection. Since ether is the electric medium, and the rigidity of the apparatus may depend upon the electric cohesion of its particles, it is not far-fetched to suppose that under those or substantially similar conditions, the flow of the ether current against the direction of the earth's travel may in some way affect that cohesion and, by condensation, shorten the apparatus.

We shall presently see that the Einstein theory involves such shortening in the direction of the earth's travel and that he therefore accepts the Lorentz explanation as in a sense true. The two thoughts, however, should not be confounded. The Lorentz theory, conceivable and even probable, is based on physical processes only, not involving in any way the revolutionizing, if not mystic, Einstein conceptions of time and space.

The Michelson apparatus was adapted only to the detection of a difference in the velocity of light against and across the current and, in each case, back again. No apparatus has yet been devised to detect the much greater difference that could be expected between the respective velocities directly with and directly against the current, that is, in the direction of the earth's motion and in the opposite direction. Einstein, however, not only rejects the plausible physical explanations of the result of the experiments in favor of his relativity theory, but also, as necessary to that theory, assumes that the result would have been the same in experiments devised to measure light's velocity in all single directions in the current. It must be remembered that the negative issue of the experiment, even if now not otherwise explicable, warrants his radical solution merely as an hypothesis to be justified only by empiric proof.

But perhaps our ignorance of the medium of light is still too dense to justify any conclusion from these experiments. If the ether, like matter, consists of ultimate particles, these particles, or some of them, may be and probably are in such a state of rest or of particular vibratory motion as to render them capable of passing, under certain conditions, through the interstices existing not only between molecules and atoms, but between electrons themselves, so that, as the earth and its atmosphere, in which the experiment is performed, rush through space, the ethereal particles or at least a substantial part of them might pass through their ultimate interstices; or, rather, the ultimate particles of earth and air might pass non-interfering particles of ether in much the way that beads move along the wire on which they are strung. If that were true, we might expect the velocity differences sought in the experiment to show themselves, but if the light-carrying part of the ether or a substantial portion thereof does not behave in the way described and is carried along with the earth, the negative result of the experiments might reasonably be expected. The Fizeau researches, later alluded to, render such conceptions plausible.

It might be added that there are other unknown factors. The experiments were conducted to discover the possibility of detecting the motion of the earth in its journey around the sun, with relation to a supposed, stationary ether. But, if the ether be stationary relative to all masses, we cannot predicate a velocity of eighteen miles per second on the part of the earth through the ether. That is its velocity about the sun. We are ignorant of what other multitude or infinitude of motions the earth in common with the sun, and the solar system in common with other possible traveling systems, *ad infinitum,* may have. The net result, both in direction and velocity, may conceivably prove entirely different from the supposed velocity of eighteen miles per second; may conceivably prove, by oppositions of motions, to be no velocity at all.

Einstein accepts the Michelson result, assumes from it the constancy of the velocity of light irrespective of the observer's motion, and from these premises sets out to deduce his special theory mathematically. Before giving the reader an idea of the mathematical formulation, we shall now undertake to show how the wonderful results of the theory may be logically reached, without the use of mathematical formulation, as necessary conclusions from the cardinal assumptions. If the velocity of light will always appear the same to all observers, whatever the direction and velocity of their own systems relative to that of the observed ray, it is perfectly evident, unless we are going to reject the validity of our logical processes, that either the measuring rod lengths, or the time unit intervals, or both, of one reference system must be different from those on another in a different state of motion.

To illustrate, let us imagine a clock placed on the sun, of identical construction with one placed on the earth, the sun being assumed to be stationary relative to the moving earth; and let us add the further assumption that the respective masses and other conditions of the earth and sun are equal, in order to prevent possible physical interference with the identical running of our two clocks. A beam of light, measured by the clock on the stationary sun, travels at the velocity of 186,000 miles per second. Let us now suppose that the earth is moving in the same direction as the beam at the velocity of 93,000 miles per second. According to the orthodox principle of relativity, to an observer on the earth the beam travels at the apparent speed of 93,000 miles per second. According to the law of the constancy of the velocity of light, however, to the observer on the earth, the light velocity is still apparently 186,000 miles per second. The only explanation, therefore, must be either that the standards of distance measurement on the moving earth are shorter than the same standards on the stationary sun, or that the earth clock moves more slowly than the identically constructed sun clock, so that when the sun clock shows the lapse of one second to a solar observer, the earth clock shows, compared with the sun clock, an advance of a shorter interval, or that both these elements are present.

Assuming both elements present, it is also apparent that the larger the supposed velocity of the earth in this illustration, the longer will be its time unit intervals and the shorter its measuring rods. If, however, the velocity of the earth's motion be assumed to attain that of light itself, it cannot be that the velocity of light would appear to the earth observer still to be 186,000 miles per second, instead of no velocity at all as demanded by orthodox ideas, unless it were true that there would be no longer any measuring rods or any time intervals or any other physical values on earth. The logical result, therefore, of the premise is not only that the difference in time and distance measurement on the relatively moving body of reference will exactly compensate for the velocity differences that one would expect to appear, but also that the velocity of light is a limiting velocity which cannot be reached or exceeded by any real body.

These results, of course, must likewise appear in more specific form with the use of mathematical formulations. We have already seen that, according to the orthodox principle of relativity, we can determine the magnitudes and directions of events, as viewed from one body of reference, in the terms from which they are viewed from another, by a simple calculation based on knowledge of the relative velocity and direction of motion of the two bodies of reference. This process is known mathematically as a transformation. The question was, therefore, presented, in view of of the supposed law of the constancy of the velocity of light, whether it was possible to devise a transformation which would be valid for that law; in other words, whether, given the time, t, and the space co-ordinates, x, y, and z, of an event with relation to one reference body, it was possible to formulate equations whereby the space co-ordinates, x', y', and z', and the time, t', of the same event, relative to another reference body, moving in relation to the first, could be determined, and in such a way as to satisfy the law of the constant velocity of light.

This problem was successfully solved by the equations of the Lorentz transformation. These equations are simple for all phenomena whose relativities are simple. The calculation of the more complex relativities is not necessary for the purpose of a comprehension of the principle of the special theory. It should never be forgotten that these equations are based on, that is to say deduced from, the cardinal assumption of the truth of the law of the constancy of the velocity of light; and it might be said in passing that they are also based on a second assumption, involved in the first, but not in the result of the Michelson experiments, that not only space intervals, but time intervals, differ, according to the relative state of motion on the body of reference on which they obtain.

We shall now illustrate in a simple way the results of these equations, disregarding as unnecessary for the purpose of illustration, precise refinements of fractions and decimals.

PROBLEM I. Let us take the case above supposed of the earth moving, relative to the stationary sun, at a velocity of 93,000 miles per second, in the same direction as a beam of light. In order, however, to simplify the mathematics of our problem through the elimination of two co-ordinates and through the identification in position of the remaining coordinate of the one system with that of the other, let us substitute for the sun our old friend, the railroad embankment, along which the beam will travel in the ether, and for the earth, our old friend, the train. This will in nowise alter the essentials of the problem. The beam then travels along the embankment in the same direction as the motion of the train (earth), according to the orthodox principle of relativity, to an observer on the train, at the apparent speed of 93,000 miles per second, while the constancy of the velocity of light demands that, even to this observer, that velocity should still appear to be 186,000 miles per second. This condition is fulfilled by the Lorentz equations as follows: The train observer would find that his mile stick was shorter on the moving train than the same mile stick on the stationary embankment (sun) and that his clock moved more slowly than the embankment clock; so that the light, which should have apparently traveled 93,000 miles at the end of a second as measured by the embankment clock and mile stick will have traveled 108,000 miles as measured by the shorter train mile stick and in a lapsed time of 58/100 of a second as measured by the slower train clock. But a velocity of 108,000 miles in 58/100 of a second is a velocity of 186,000 miles in one second.

The Lorentz transformation equations show an interesting consistency in all their applications, and also some curious results, with which we can afford to let the mathematicians wrestle. We shall merely indicate them. The transformations show that a unit measuring rod on the train, in the case supposed, becomes approximately 866/1000 of its length on the embankment. As the train mile is therefore only 866/1000 of an embankment mile, the 93,000 miles of relative velocity of the light as judged from the embankment becomes by division by that fraction, the 108,000 miles above posited, as judged on the train. The transformations, however, produce also the result that a train second on a clock at the train origin, that is, at the point originally coincident with the embankment observer and the origin of the flash, and not at the point of the event, which in this case is the end of the light travel on the completion of an embankment second, will be 1000/866 of an embankment second. The *lapsed* time for the completion of the event, however, turns out to be 58/100 of a train second, the equivalent of about two-thirds of a second on the embankment. This curious result is due not only to the difference in time unit values on the two systems, but also to the initial inability, above noted, to synchronize their clocks.

Problem 2. The same calculation may be made with reference to the supposed case of the train (earth) traveling at the velocity of 93,000 miles per second in a direction opposite to that of the ray of light. In this case, the relative velocity of the light to the train, according to orthodox standards, would be 279,000 miles per second, but the mile stick on the train would still be 866/1000 of that on the embankment (sun), so that the apparent 279,000 miles becomes 322,000 train miles. The train duration for this travel, transformed by the equations, becomes 173/100 of the train second. A velocity of 322,000 miles in 173/100 of a second becomes again a velocity of 186,000 miles in one second. Since the constancy of the velocity of light was assumed as the basis of the equations, the results of the two problems are not surprising.

PROBLEM 3. The equations demonstrate, however, not only that lengths and durations on a moving body of reference will be less than on a relatively stationary body, *but that they will likewise be less on the stationary body as compared with those on the first-mentioned body of reference,* since in that case it will be the supposed stationary body which has the motion relative to the other. We shall have more to say of this mystic result later.

The equations show, then, in mathematical form, that a clock in motion will go more slowly than the same clock at rest; that the time units of the moving clock will be longer; that at the velocity of light, clocks, and in fact all motion will stop, there will be no time at all, and measuring lengths and bodies will disappear; and that the velocity of light will be the same for all systems, whatever their state of motion.

The foregoing are some of the results of the special theory of relativity, which is the restricted principle of relativity corrected by the additional assumed fact of the constancy of the velocity of light. Professor Einstein, himself, formulates the special theory of relativity as follows:

> "Every general law of nature must be so constituted that it is transformed into a law of exactly the same form when, instead of the space-time variables, x, y, z, t of the original co-ordinate system K, we introduce new space-time variables x', y', z', t' of a co-ordinate system K'. In this connection the relation between the ordinary and the accented magnitudes is given by the Lorentz transformation. Or, in brief: General laws of nature are co-variant with respect to Lorentz transformations."

The beauty of the theory is that, complex as it seems at first, it really amounts, if sound, to a simplification. If it be true that there is no constancy in time or distance units, that in different places or conditions of the universe such units differ, confusion must result in all sufficiently refined calculations. The theory pretends to resolve such confusion by a transformation which, whatever the condition of the observer, will automatically show the absolute truth. Under the method of that transformation the velocity of light becomes the same for all observers. It must not be forgotten, however, that this constancy, first assumed, is the basis of all that follows, including its own proof. The Lorentz transformation equations prove that constancy because it was the premise on which they were built up. Moreover that constancy is merely an hypothesis suggested by the result of the Michelson experiments and not any necessary conclusion therefrom or, as we shall later seek to demonstrate, from any independent logical process.

CHAPTER VI

SPACE, TIME, MOTION, MASS AND ENERGY ACCORDING TO THE SPECIAL THEORY

THE Lorentz theory of a physical contraction as the explanation of the results of the Michelson experiments requires, of course, that this contraction in the direction of motion should be fully compensatory, that is, precisely in the ratio inverse to the calculated ratio between the time required for the travel, in one arm of the apparatus, of the light between the two mirrors in the direction of the earth's travel, and that, in another arm, for such travel in the perpendicular direction. As we have already remarked, however, the experiments have determined nothing in relation to the question whether the velocity of light against the current, that is, in the precise direction of the earth's travel and in that direction only, not forward and backward again, would turn out the same as its velocity in the opposite or reverse direction, and again in that direction only. According to hitherto accepted standards, the ratio of difference ought to be enormously greater than in the case actually involved in the experiment. If the velocity should turn out to be the same, that incredible result could not possibly be explained on the Lorentz theory of a physical contraction, since both arms of the apparatus would point in the same direction, that of the earth's motion, and the compensation required would be too immense and inconsistent in application.

If a ray of light were sent in the direction of the earth's travel through a tube affixed to the earth, its apparent velocity as compared with that on a relatively stationary system, ought to be reduced, to an earth observer, by the amount of the earth's velocity, and when sent in the opposite direction, again through a similar tube of equal length, its apparent velocity should be correspondingly increased, the difference between the two apparent velocities, therefore, being equal to twice the earth's velocity. It is like the case of a swimmer traveling against a current and of an equal swimmer traveling with the same current. If it were possible, by experiment, to establish that the two velocities would still be the same to the earth observer, it is apparent that that result could not be explained by the unknown contraction of the tubes, because that contraction, while it would increase the apparent velocity in the forward direction by the amount of the earth's velocity, would have to reduce the velocity in the rear tube by the same amount in order to produce the miraculous result, while the contraction of the rear tube would naturally have just the opposite effect.

Nevertheless, Einstein assumed that both these velocities would still turn out the same. As we have seen, this result cannot be explained on the theory of the Lorentz contraction. A little figuring will show that it can be explained, and still on classic grounds, by the non-synchronization of the clocks on the moving earth and those on the hypothetic relatively stationary system. On the other hand, that explanation can have nothing to do with the result of the Michelson experiment, where the comparison was between the time taken for light travel from a point out and back again in the direction of the earth's travel and from the same point out and back again in a perpendicular direction, an apparently equal distance; because in that experiment the difference in time is taken in a way equivalent to the reading of clocks on the same system, even by the reading of the same clock.

Einstein was, therefore, driven to test the revolutionary hypothesis that no difference can be detected between the various velocities of light sent in different directions relative to the motion of the observing system, and whether to and fro or in single directions only, because the relative velocity of the system on which the experiment is made not only affects dimensions in space in the direction of travel, as is assumed by the Lorentz contraction theory, but also dimensions in time; and in a compensatory way. Anticipating somewhat, and in order to clarify this view, we offer the following illustration.

Suppose the earth's velocity to be 161,000 miles per second. In that case the exit of our rear tube should, according to orthodox standards, advance toward the approaching light at the velocity of 161,000 miles per second, so that the light would be approaching that exit at a velocity equivalent to 347,000 miles per second. That is to say, picturing the matter as though the earth were stationary and the ether in motion, it would be the case of the light adding its velocity to that of the current in which it swims. On the other hand, the exit of the front tube would be moving at the rate of 161,000 miles per second away from the pursuing light, with the result that the latter's apparent net velocity in its journey through the tube would be reduced to 25,000 miles per second, or about 1/14 of the velocity of the ray through the rear tube.

We may imagine our earth observer so astounded when he finds these two velocities nevertheless to be the same that he sends with each ray on its journey through its tube a wizard observer equipped with magic apparatus capable of measurement of and comparison between earth units of space and time measurement and those of imaginary observers embedded in the hypothetic stationary ether at all material points, if such observers be possible, or of observers on relatively stationary and unquestionably possible, physical systems.

When the two journeys are completed and these wizards emerge from their respective tubes of supposed equal length, Mr. A. from the front tube, and Mr. B from the rear one, to discuss the matter with the earth observer, they say:

"We are not surprised that you expected Mr. B to emerge long before Mr. A, but that is because you have, and can have, no conception of what the velocity of the earth and tubes relative to the relatively stationary physical systems, if not to your supposed absolutely stationary ether, did to the space medium in which we and they traveled and to your standards of measurement as compared with those of the other observers. The medium is not as you think it at all; its reality is inconceivable to you. You call it space because you have no other word for it, and because that is the form in which your circumstances compel your mentality to react to the phenomena involved as you perceive them. We know better. The 'space' in which we traveled has four dimensions and one of them is time. We might, therefore, call it space-time. The way we actually traveled through it you could never picture. From your point of view, as well as from that of the imaginary ether observers and the others, we were traveling in straight space lines during time. But we could see that your standards and theirs, for the measurement of this imaginary space and time, were quite different. This was due to your great velocity relative to them. They would have seen each of us traveling at 186,000 miles per second and disregarded your motion, though of course, considering that motion, they would have agreed with you that we should have emerged from the tubes at different moments. So far as you are concerned, we were able to see the effect of that motion and to separate and determine in language clear to your point of view the apparent space and time dimensions of the continuum in which we traveled. Even on your own standards you should have remembered, which apparently you have not done, that, though you believe yourselves to have considered your own motion, you have not fully done so, in supposing your clocks to be in harmony with those of the stationary observers. Your cardinal error, however, consists in supposing your mile and second to be the same as those of all other observers, which is far from the fact. Really, there is no ether and no ether observer. There is no system in absolute rest from which to transform time and space values, as you think. Those values for you differ in relation to those on another physical system, which you may regard as in motion relative to yours, but which those on it regard as at rest relative to your motion. The reason, therefore, that our velocities turned out to be the same, that is to say, as though there were no current, or, to put it more precisely, the reason that we emerged from the tubes at the same instant in apparent defiance of the supposed difference of our respective journeys by reason of the tube motion, will be to some extent comprehensible to you when we tell you that A found that the tube distance traveled in one of your supposedly standard seconds was not the difference between his velocity and the tube's, as you supposed, but twice as much, and that your second was not as you supposed it the same as the ether second but one-half as much, and making the necessary allowance for synchronization, the earth clock being earlier, approximately one-quarter, so that you having expected his apparent velocity to be 25,000 supposed standard miles in one supposed standard second, he found it to be 50,000 of your miles in a little less than 27/100 of your second, which is the equivalent of about 186,000 of your miles in one of your seconds. In other words, he gained on the tube, not at 25,000 miles per second, but at 186,000 miles per second. On the other hand, B found that the space dimension traveled in one of the supposed standard seconds was not the sum of his velocity and the tube's, as you supposed, but twice as much, and that your second was not, as you supposed it, the same as the ether second, but again one-half, or, properly synchronized, the earth clock being later, approximately four times as great, so that you having expected his apparent velocity to be 347,000 supposed standard miles in one supposed standard second, he found it to be 694,000 of your miles in 373/100 of your second, which is the equivalent of about 186,000 of your miles in one of your seconds. He gained on the tube, not at 347,000 miles per second, but at 186,000 miles per second. In both cases it turned out *as though the tubes were not moving at all*. It was thus natural that we should complete our journeys at the same instant. We have heard of Professor Einstein who has predicted these results as a matter of reasoning, but it is well you selected us wizards instead of men as observers because even at your velocity of 161,000 miles per second, as indeed the Professor seems to have predicted, you were not able to make the explanation, since even that great velocity did not enable you directly to perceive the fourth dimension."

Now these wizards might have said, as later indicated in Chapter VII, that there was no tube or earth motion at all, which is the same as saying that they found no ether current either to assist or retard their own velocities. This would have dispensed with all the figuring. They probably saw, however, that such an explanation would have been confusing in relation to the earth obsession of a fixed observer in the ether or somewhere else who would be supposed to have actually perceived that motion, and therefore adopted that point of view. But even from that angle there is something in their reasoning, wizards though they be, not to the writer's liking. This we shall note in Chapter VIII.

According to the Lorentz transformation, the length of a unit measuring rod on one reference body moving in relation to another, as determined by an observer on this other, is $\sqrt{1-\frac{v^2}{c^2}}$ and the duration of the time unit on the relatively moving reference body similarly determined is $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, v standing for the apparent or relative velocity of the one reference body in relation to the other, and c for the velocity of light. It is apparent, therefore, that unless the quantity v is very large, as compared with the quantity c, the difference in length and time units will be so minute as to escape observation.

Even at the velocity of the earth, 18 miles a second, the fraction to be deducted from the unit 1 will be the square of 18 divided by the square of 186,000, which, for practical purposes, leaves the unit length, both of the measuring rod and of the time interval, the same. The special theory, therefore, as we have no ordinary experience of velocities comparable with that of light, has no ordinary relation to human affairs. In fact, the only velocities now known by men which in any wise approach that of light are those connected with electrical phenomena and electronic motion.

The special theory, therefore, is of interest chiefly in relation to the new cosmological views entailed by it. The assumption as true of the miraculous destructions, outlined in the last chapter, of time and space magnitudes, certainly requires a new formulation of our notions of motion, space and time, which we shall now proceed to develop.

We do not see things instantaneously, as the ancients supposed, for, in spite of the enormous velocity of light relative to that of ordinary earthly velocities, it takes a perfectly definite interval for the light, whereby we perceive an object, to travel from it to our eyes. Therefore, even according to old standards, in calculating the motional magnitudes of an observed object, it was theoretically proper, though practically negligible, to take into consideration the velocity of light. This consideration, however, involves a very simple calculation in accordance with the classic principle, in no way affects natural laws, and does not enter into the formulation of the Lorentz equations.

To clear the ground further, it might as well also be remarked at this point that, in the statement of the propositions of the Einstein theory, it is never to be understood that determinations of length and time units on a relatively moving reference body by an observer on a relatively stationary one have to do with defects of human vision. In all these propositions and the accompanying illustrations, it must be assumed, in order to comprehend the points adduced, that the vision is of a telescopically adequate character, the apparent resulting paradoxes, if such there be, not being due to defects of vision, but to those natural phenomena, if there be any, that are the discoveries of the special theory.

As heretofore remarked, the Lorentz transformation equations are derived from the premise (1) of the law of the constancy of the velocity of light. This premise, as also previously remarked, is inexplicable except on the assumption (2) that that strange law must be due to variations *both* in length and time units as between two relatively moving systems, which means (3) the transformations of the time and co-ordinates of one system into those of another through multiplication or division by quantities based on a complex but constant ratio between v, the velocity of one system relative to the other, and c, the velocity of light. The cardinal assumption (1), then, is that on which the present theory and our discussion of it turns. It should be noted that that assumption applies to the constant velocity, not of a body or mass, but of a ray of light as determined from two relatively moving systems.

This last fact neglected, it might seem surprising that length and time units on two different systems should differ in a ratio determined by the difference between the relative velocity of the systems and the velocity of light. One might wonder in what way, logically, the velocity of light was concerned with this apparently unrelated matter. That motion of a system should change time and length values is verbally intelligible, but why in degrees dependent on the velocity of light? The answer lies, of course, in the premise. If one is to assume, and solely on the basis of the Michelson experiment, that light and light alone is of constant velocity from whatever system its velocity be measured, it is not surprising that the resulting formulæ give the different length and time units of the two systems quantitatively in terms of the velocity of light, though light itself or its velocity is not a part of any particular problem presented.

In other words, if light will always show the same velocity in relation to any state of motion in the co-ordinate system selected, a phenomenon caused by a precisely compensating difference in measuring values, it is not surprising that when these values change because of the relative velocity of two systems, they must do so according to some relation between that velocity and the velocity of light. The only natural reason for coming to such a conclusion is, perhaps, to be found in that other surprising conclusion from the

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premise,—that the velocity of light is a limiting velocity which can be exceeded by no real body. That might afford some logical basis for the law that as moving bodies approach that limiting velocity, their length and time units must be accordingly affected. It should not be overlooked, however, that if lesser ethereal velocities than that of light should also prove to exhibit this phenomenon of apparent constancy, or if any velocity exceeding that of light should be discovered, there would be a surprising mix-up.

Professor Einstein, however, evidently assumes the intelligence of his reader equal to the task of finding some reason for the application of his equations, containing, as a quantity, *c,* the velocity of light, to all phenomena of motion within the limits of his theory, for, after the statement of the theory, he leaps boldly and without more ado, into a mathematical formulation, based on the Lorentz equations containing *c,* of the phenomena resulting from the addition of energy to moving bodies. Assuming, however, the validity of this apparent leap over the logical chasm, which we have, so inadequately, just attempted to fill in, we have, as the mathematical result, the proposition that a moving body as it loses in length gains in mass until, at the velocity of light, its mass becomes infinite.

This might mean, to go back to the problem of Chapter I in relation to the moving billiard ball, that that which is added to a body to set it in motion is really an entity which involves a contraction in length and an increase in mass, too small, at earthly velocities, to be noticeable, but none the less real. This mathematical result, if authentic, carries the further profound revelation that, since at the moment of the body's disappearance as a real body, when it reaches the velocity of light, its mass has become an infinity, that which has been known to us as a real body, in disappearing as such, has been received into the vast ocean of infinite energies.

Thus, the inertial mass of bodies becomes a measure of their energy, and from this Professor Einstein deduces that the law of the conservation of mass, that is to say, the law of the indestructibility of matter, becomes identical with the law of the conservation or indestructibility of energy. In this connection, however, it might be remarked that the Einstein theory was not needed for what has been a common conclusion of philosophers for many years. They have long known that, no less than energy manifested in molar or molecular motion, matter, in its resisting and extended essence, the "something-there-ness," termed substance, is analyzable into a manifestation of force. Matter and energy are one.

Before proceeding further with the discussion, it is well to note here an example of the danger of relying too much on mathematical abstraction, afforded by the formulation of Einstein's application of his transformations to problems of additions of energy to moving masses. He tells us that the kinetic energy of a mass, *m,* is no longer given by the expression $m\frac{v^2}{2}$, but, according to the Lorentz transformation, by the expression $\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$, where v is the velocity of m, and c again the velocity of light. In other words, the energy, for all ordinary velocities, becomes mc^2, meaning the mass multiplied by the square of the velocity of light. This is certainly a meaningless expression unless the c value be eliminated by making it unity, and is so felt to be by the Professor, who says:

> "The first term mc^2 does not contain the velocity, and requires no consideration if we are only dealing with the question as to how the energy of a point-mass depends on the velocity. We shall speak of its essential significance later."

Later, he shows that a body moving with the velocity, v, which absorbs an amount of energy, E, without change of velocity, has, after the absorption, an amount of energy represented by the expression $\dfrac{mc^2 + E}{\sqrt{1 - \dfrac{v^2}{c^2}}}$. At ordinary velocities the expression becomes $mc^2 + E$, in other words its original energy mc^2 plus the added energy E, so that the Professor remarks:

> "We see that the expression mc^2, which has hitherto attracted our attention, is nothing else than the energy possessed by the body before it absorbed the energy, E."

In other words, the result of the mathematical formulation, however true the quantitative relations expressed may be, is to give a meaningless expression for the original energy.

Mathematical formulation, however, is not needed to prove the fact that, the special theory being true, the increasing velocity of a body will mean an increase of mass. The momenta of two equal bodies propelled with equal velocities from each of two relatively moving systems in a direction perpendicular to that of relative travel are, of course, the same. The velocity of each body need not be added to that of its system since its only velocity relative to that system is the velocity of its propulsion from that system. The momentum of a body is the product of its mass by its velocity. Now, as judged from either one of the two systems regarded as relatively stationary, the time unit on the other system is longer, so that the velocity of the propelled body on either system as viewed from the other is less. The momenta, however, being the same, the mass must have become greater.

To illustrate this further, and to draw from it certain most startling conclusions, the reader will pardon the introduction of one further simple mathematical formulation. According to classic mechanics, if one velocity be added to another, the body will move at a velocity equal to the two. This is the result reached in Chapter IV, in relation to the problem of the man walking in the train. If, therefore, a body is accelerated by a force constantly acting, its velocity should constantly increase by the exact amount of the acceleration. These propositions, however, are not true according to the special theory.

A body's velocity in any system is the distance traveled by it in the time unit of that system. According to the special theory, however, both the distances and the time units vary with each increase in the velocity, and in such a way that the aggregate velocity will be less than figured by classic standards. This diminution will be more as the additional velocities are larger and larger. The precise mathematical formulation of this surprising fact is as follows:

$$w = \frac{v + u}{1 + \frac{vu}{c^2}}$$

where w is the final velocity, v, the original velocity, u, the added velocity, and c the velocity of light.

If anyone will take this equation and make the added velocity u, the same as the velocity of light, c, he will find that the equation will work out so that w, the final velocity, will be c, the velocity of light, even though c is the quantity that has been added to the original velocity v. In other words, we again have here the demonstration that, according to the theory, there can be no velocity exceeding that of light, and that even if a body moving with a given velocity have added to it the velocity of light, the resulting velocity will still be that of light.

Therefore, even if the velocity of a body be constantly accelerated, since the velocity does not increase in the proportion which we would naturally assume, and, when the velocity of light is attained, ceases to increase further, we must draw the conclusion that the reason is an increasing resistance in the body, that is to say, an increasing mass or inertia. When the velocity of light is reached, the mass has become infinite, and that velocity cannot be exceeded because it would take an infinite force to overcome the infinite resistance.

No sooner do we imagine, on the consideration of such conceptions, based on the decrease of length units and enlargement of time units on moving bodies, that we are reaching the true Einstein cosmology, when we are recalled to a possible contrary conclusion, by remembering, with a shock, an important, perhaps the most important feature of the theory, which seems, at least on first examination, to be inconsistent with these conceptions. We remind the reader of Problem 3 of the last chapter, where, as a result of the Lorentz transformation, it was determined that not only do the length units become shorter and time units longer on body B, as compared with the same units on body A, A and B being two relatively moving bodies, but, likewise, that length units become shorter and time units longer on body A as compared with the same units on body B. Now, if one is told that on body A a length *is,* not *appears,* shorter as compared with the same length on body B, he would naturally expect that on body B the length would be longer; and the same with the respective time unit measurements. If a less illustrious name than that of Professor Einstein were behind the contrary assertion it would be deemed a *reductio ad absurdum.*

Nevertheless, he directly asserts it. After stating that the length of a meter rod, moving in the direction of its length with a velocity v, is the quantity given earlier in this chapter (p. 117) when moving along the x' axis of reference body K', he says:

> "If, on the contrary, we had considered a meter rod at rest in the x-axis with respect to K, then we would have found that the length of the rod as judged from K' would have been $\sqrt{1 - v^2/c^2}$ [the same as found for the

meter rod on the x' axis of K'] ; this is quite in accordance with the principle of relativity which forms the basis of our considerations."

Moreover, this strange phenomenon is the mathematical outcome of the Lorentz equations. Let us take again Problem 1 of the last chapter, this time reversing, so that our calculations shall not be made on the embankment, but on the train. In the last chapter we showed the determination on the embankment, by the equations, of certain measuring magnitudes as they were on the train as compared with those on the embankment, the train being supposed to move along the stationary embankment at a velocity of 93,000 miles an embankment second, the problem being the velocity, as figured from the train, of a ray of light moving in the same direction at a velocity of 186,000 miles per second as viewed from the embankment. We now have the reverse problem of determining that velocity as viewed from the embankment, but as worked out by the relatively stationary train observer, who regards the embankment as moving in relation to him at the velocity of 93,000 miles per second, in the opposite direction to that of the formerly relatively moving train. On orthodox standards, the relative velocity of the ray of light, taking into consideration the opposite motion of the embankment, as the train observer must do in taking the embankment point of view, is now 186,000 miles per second from that point of view, since it is evident that to the apparent velocity of the embankment from the train in one direction must be added the apparent velocity of the light from the train, in the other. But as, according to the special theory, the mile of the moving embankment is no longer the same, this result cannot be correct.

The error is partly rectified by the Lorentz equations in the following way: The measuring rod upon the moving embankment is, as compared with that on the stationary train, 866/1000, precisely the same as the measuring rod on the train, as compared with that on the embankment in the reverse problem; and the 186,000 miles of the present problem thus becomes 215,000 miles. But in the same way, the clock on the embankment will now appear to go slower as compared with the train clock, in precisely the same degree as the train clock in the other problem appeared to go slower as compared with that on the embankment, the interval of a second on the embankment turning out 1000/866 of that on the train. A velocity of 215,000 embankment miles in 1000/866 of a train second is 186,000 *embankment* miles in one *train* second.

Now this is no solution of the problem, and if the equations had been applied to find the lapsed time on the embankment in the case just put, the result would have been still more incongruous. The reason is that the problem as just solved was purposely, for illustrative purposes and for the sake of simplicity, not posited accurately. We began by talking of a second, as though its duration were constant, and of a velocity of light, as viewed by the train, of 93,000 miles a second, though the theory assumes its constant velocity of double that amount from any viewpoint. We were entirely forgetful of the difficult and complex bases of the theory. If we had forgotten our prejudices and viewed the train as truly stationary, the light, in a second, *by its clock,* would not have appeared to travel 93,000 miles as we stated, but 186,000 miles in the opposite direction to that of the embankment's motion of 93,000 miles a second by the train clock, so that the light would have appeared to the train observer to have traveled 279,000 miles from the embankment observer. This is again Problem 2 of the last chapter with the same, and therefore a perfectly harmonious result. The following is a simpler way to solve the reverse of Problem 1, from the standpoint of the special theory.

We should have assumed the result of Problem 1, that in 58/100 of a second on the train the ray will have traveled 108,000 train miles. According to the equations, however, if the train be regarded as at rest and the embankment as moving, the lapsed time on the embankment, determined by multiplying 58/100 by 1000/866, is about two-thirds of an embankment second. In the same way the 108,000 miles, as viewed from the train, becomes, through division by 866/1000, 124,000 embankment miles. But a velocity of 124,000 miles in two-thirds of a second becomes again 186,000 miles in one second.

Without wearying the reader with further mathematics, it may be said that we might just as well have taken the reverse of Problem 2 of the last chapter. That and every other variation that may be made on the basis of the Lorentz equations will always yield the surprising demonstration that, as viewed from the train, length units are shorter and time units longer on the embankment than on the train, and that as viewed from the embankment, length units are again shorter and time units longer on the train than on the embankment.

This leads us to the inquiry whether the results of the Einstein theory are matters of illusion only, or of actuality. The Professor certainly does not regard them as illusions, as is abundantly demonstrated, not only by his precise language, but by the further development of his views, as will presently appear. He tells us that "the rigid rod is thus shorter when in motion than when at rest, and the more quickly it is moving, the shorter is the rod." He tells us that, as the result of the Lorentz transformations, "if a body takes up an amount of energy, then its inertial mass increases" by the amount of that energy divided by the square of the velocity of light. He tells us that "as a consequence of its motion the clock goes more slowly than when at rest."

These are not statements of apparent, but of actual, results. The proposition, however, that a time unit is longer and a length unit shorter and a mass of initial equality greater on a moving body A than on a relatively stationary one B, coupled with the further proposition that the same results follow from the A point of view, when B is regarded as the moving body, compels the logical conclusion that the differences are apparent only, unless some profound truth underlies these apparently conflicting propositions, so subtle as at first to escape us.

This leads us, before we go further in the analysis, to a review of our whole previous judgment of the Einstein reasoning, in order to test whether or not we have misunderstood him; but before proceeding with that test in the next chapter, we ought to obtain, if possible, a clear idea of the sense in which, in the discussion, such terms as "actual," "real," "inherent" and "absolute," as distinguished from such as "relative" and "apparent" are used. In Chapters I and II it was shown that the metaphysician or philosopher analyzes phenomena and traces them, inductively, as effects, until he reaches an ultimate, absolute and infinite First Cause, inconceivable by finite intelligence, and with respect to which further reasoning must fail. It was shown that after this point is reached, there can be only metaphysical or theological speculation. The "absolute" in that sense is beyond us.

Mental states, which alone consciousness can know, were posited as phenomenal, apparent or relative, while the underlying reality, of whose existence we are inductively convinced, but which, again, is beyond us, was posited as the actual, real or absolute. We showed, for example, that substance is presented to mind in terms of muscular resistance and of extension determined by traveling touch and by movement of the ocular muscles in taking in the picture visually. The tactual, muscular and visual mental impressions constitute our knowledge; but it is *relative* only, in the sense that the underlying essence is alone the actual.

So with motion. From our earliest experience, we become acquainted with phenomena or appearances, that is to say, with sensations, of varying degrees of strain and shock. These sensations, such, for example, as the strain of lifting, are subjective, mental. They are not in the object. The experiences of the developing mind gradually build up complexes of matter extended and moving in space and time, from these primal sense impressions of strain and shock and muscular tension, associated with visual and tactual mental pictures. The mental operation turns out, inductively, to be the effect of something external to the *ego*. The latter *reaches* objects, converts them, for example, from visual into tactual experiences, only by muscular effort. The orange of Chapter I is a complex soon attained in its totality, but objects at unreachable distances complete the conception of external vastness, borne also of our mental inability to limit the theoretic, infinite potentialities of motion beyond any particular place. The child may see, but not touch the moon. The visual and tactual impressions, the strains and shocks, constitute our only direct knowledge of those modes of being termed matter, force, space, time and motion. Such knowledge is, therefore, relative only. If we could know what those modes of being are inherently, such knowledge would be real, actual, absolute.

In the present discussion, we are compelled, for lack of a fuller terminology, to use these terms in quite a different meaning. Einstein speaks of motions, and we must follow him, as being actual or absolute, as distinguished from being only relative or apparent, in a physical or phenomenal sense; at least we so understand him, though sometimes, it must be confessed, he confuses the discussion by a curious mixture of the metaphysical use. His concern seems to be with relativity as against actuality *within* the phenomenal or relative world, with illusion as against reality.

In the discussion, for example, of his discovery of a difference in measurement units dependent on the velocity of the system, our inquiry must be whether he means this difference as phenomenally actual or illusory, even though the terms used are the same as in metaphysical discussion, "absolute" or "real" or "actual," as distinguished from "relative" or "apparent." In the physical sense, we say that the motion of our train along its embankment is real, that of the embankment illusory and apparent only. We know nothing metaphysically of the real, actual or absolute motion of either. When Einstein says neither has actual motion, but each has only relative motion as regards the other, the inquiry is, whether he is speaking of actualities as distinguished from illusions in the every-day sense of the terms, and in the relative or phenomenal world only. We shall resume the analysis of these difficult conceptions in the next chapter, after first reviewing there the Einstein reasoning, as above suggested.

It will suffice here to say that if he discusses "actual" or "absolute" motion in the metaphysical sense, his discussion becomes no longer scientific; he has attempted to penetrate the noumenal world. He is speculating on things of which men, by the very nature of their thinking apparatus, must remain, forever, intellectually ignorant.

CHAPTER VII

THE ORIGIN OF THE SPECIAL THEORY MORE CLOSELY EXAMINED

We have heretofore assumed Einstein's "Restricted Principle of Relativity" to mean the ordinary relativity prevailing before his time, to wit, the principle that while we can only know motion relatively through change in position of bodies, the motional characteristics of an event can be ascertained by simple calculation in relation to either or any of two or more reference bodies or co-ordinate systems if those characteristics, in relation to one, and of the systems in relation to each other, are known. We have further assumed that the result of the Michelson experiments as accepted and interpreted by Einstein seemed in conflict with the restricted principle until the reconciliation was effected by the special theory, which substitutes in the restricted principle, for the classic transformations, the Lorentz transformations.

These assumptions were made since they

seemed to the writer the only logical basis for the development of the special theory. We shall now point out that there is much in the Professor's treatise of an obscure nature, which seems to denote an initial vacillation on his part between these ideas and another which he seems, however, finally to adopt. That other is that, given the restricted principle in the classic sense and the law of the constancy of the velocity of light, not in the sense in which we have been using that expression, but in the primary and also classic sense explained in Chapter V, the special theory may be logically deduced. In other words, the special theory is contained in the restricted principle, that is, follows from the relativity of motion. The results of the Michelson experiment and the ensuing hypothesis of the constancy of the velocity of light independent of the motion of the observer, according to this view, might have been logically deduced and predicted from the two cardinal premises:

(1) No one can detect his own actual, uniform, rectilinear motion, but only his motion relative to some other body.

(2) The velocity of light in a vacuum is constant *whatever the velocity of its source.*

For convenience, we shall sometimes refer to this constancy in the sense just indicated, as the primary law, and to the constancy of the velocity of light independent of the motion of the observer, as the secondary law.

We shall now analyze the early portion of the Professor's book, with the idea of showing the obscurity to which we have called attention, and the inconsistency that grows out of it in such a way as to lead finally to his adoption of the second view above outlined. We shall then examine that view, coming to the conclusion that it is unsound and that the special theory must stand alone upon the acceptance of the result of the Michelson experiments and the hypothesis adopted by Einstein to explain that result. We shall begin the next chapter with a closer examination than we have yet made of the results of the special theory, in the light of the review we are about to make of the Professor's ideas in this chapter.

We shall there inquire into the possibility of devising some conception or picture of the new world necessitated by the theory. The Professor, with a praiseworthy conservatism, confines himself largely to his mathematical results and does little to aid in their interpretation. Some interpretation, however, is not only highly desirable from the philosophic standpoint, but is necessitated if Einstein's contribution to knowledge is to be properly appreciated.

In the early part of the treatise the Professor, after setting forth as an example of ordinary relativity of motion, the case of the stone dropped from the train, says:

> "Let us imagine a raven flying through the air in such a manner that its motion, as observed from the embankment, is uniform and in a straight line. If we were to observe the flying raven from the moving railway carriage, we should find that the motion of the raven would be one of different velocity and direction, but that it would still be uniform and in a straight line. Expressed in an abstract manner we may say: If a mass m is moving uniformly in a straight line with respect to a co-ordinate system K, then it will also be moving uniformly and in a straight line relative to a second co-ordinate system K', provided that the latter is executing a uniform translatory motion with respect to K. . . .
>
> "We advance a step farther in our generalisation when we express the tenet thus: If, relative to K, K' is a uniformly moving co-ordinate system devoid of rotation, then natural phenomena run their course with respect to K' according to exactly the same general laws as with respect to K. This statement is called the *principle of relativity* (in the restricted sense)."

Thus, the restricted principle is shown to be based on such ordinary relativity as that the raven, while observed from the train to be moving at a different velocity and direction than as viewed from the embankment, is still moving at a uniform rate and in a straight line. Not a word has been said, up to this point, opposing the restricted principle, in any way, to classic ideas. The meaning is that, under that principle and by simple calculations, the direction and velocity of the bird as viewed from the train can be easily transformed into its direction and velocity as viewed from the embankment.

The Professor goes on to state that certain recent developments in electrodynamics and optics render it possible that the restricted principle does not account for all the laws of nature. We understood him to refer here to such researches as those which have tended to show that moving electrons gain in mass and lose in length, and to the Michelson experiments accepted by him as showing the secondary law above defined. If the velocity of light is constant to every observer, whatever his own state of motion, some addition to the restricted principle is certainly needed to account for that astounding exception to the laws of relativity and of nature as heretofore understood. We have, therefore, understood the

Professor to proceed to justify the restricted principle in order not to reject it, but to add to it his space-time theory, which is to reconcile it with the secondary law.

He makes this justification upon two grounds. The first is the accuracy of the principle in the general mechanical field including the motions of the heavenly bodies, with which he states the principle supplies us "with a delicacy of detail little short of wonderful." The second argument runs somewhat as follows: If the restricted principle does not hold, then it must follow that the laws of nature must be entirely different as viewed from one system than as viewed from another in a different state of motion, that is to say, must be non-transformable as between the two systems by simple calculation. In other words, if the restricted principle were not true, we should have to look for some special or unique body of reference which we could regard as at absolute rest and from which the laws of nature could be stated with unique simplicity, while they must appear different and non-transformable as viewed from a system in motion relative to the unique one. He goes on to say, however, that no such difference in the laws of nature, no such non-transformability, has ever been discovered by reason of relative differences of motion in the observer. He takes the earth as an example by reason of the change in its direction in every instant of its travel about the sun, and concludes the argument, in Section V of his book, in the following words:

> "If the (restricted) principle of relativity were not valid we should therefore expect that the direction of motion of the earth at any moment would enter into the laws of nature, and also that physical systems in their behaviour would be dependent on the orientation in space with respect to the earth. . . . However, the most careful observations have never revealed such anisotropic properties in terrestrial physical space, *i.e.,* a physical non-equivalence of different directions. This is a very powerful argument in favour of the principle of relativity."

The next step in the Professor's exposition is to call attention to the primary law of light. He says that "as every child at school knows," light is propagated in empty space at a constant velocity of 186,000 miles per second, independent of the velocity of the body emitting the light. This, it must be admitted, is a statement of the primary law, as shown not only by the clear language used, but by the statement that every child at school knows it, because every child certainly does not know the secondary law. His readers must, therefore, be somewhat confused when, without any mention of the secondary law, he proceeds to state that the restricted principle is apparently contradicted when a classic transformation shows that a ray of light will appear to have different velocities dependent upon the motions, relative to it, of the respective observers. He has not, up to this point, in any way suggested, and, in fact, never does suggest, throughout his treatise, the far from self-evident proposition that from the premise of the relativity of motion, that is, the inability to detect one's own motion except in relation to other bodies, in conjunction with the premise of the primary law of light, the secondary law must follow. In the absence of any such suggestion, it is entirely impossible to follow his argument that, because light will develop different velocities to different observers, there is an apparent contradiction of the restricted principle.

He does make that argument, however, and immediately follows it with this language:

"At this juncture the theory of relativity entered the arena. As a result of an analysis of the physical conceptions of time and space, it becomes evident that *in reality there is not the least incompatibility between the principle of relativity and the law of propagation of light,* and that by systematically holding fast to both these laws a logically rigid theory could be arrived at. This theory has been called the *special theory of relativity* to distinguish it from the extended theory with which we shall deal later."

It is on this basis that our initial assumption was made that the Lorentz transformations were added to the restricted principle, thus converting it into the special theory, for the purpose of accounting for what we designate here as the secondary law of light. We find the argument to be reasonably clear, to the effect that the relative motions of systems do not interfere with the transformability of motional magnitudes as viewed from one system into those viewed from another; and that, if the restricted principle were not true, the laws of nature would not be the same, viewed, for example, from the moving earth, as from the relatively stationary sun, but that since they are the same the restricted principle must be true.

We are first disturbed in this view of the argument, however, when later, after a full exposition of the special theory and while discussing the evidences of its truth, the Professor tells us that his reference, in the matter just quoted from Section V of his book, to the failure to discover any physical non-equivalence of different directions, is to the Michelson experiments. He says:

> "The second class of facts to which we have alluded has reference to the question whether or not the motion of the earth in space can be made perceptible in terrestrial experiments. We have already remarked in Section V that all attempts of this nature led to a negative result. Before the theory of relativity was put forward, it was difficult to become reconciled to this negative result."

The Professor goes on to intimate that this negative result, before the special theory, could only be interpreted physically as a seeming exception to the laws of nature as previously understood, since one reference body, such as the sun, was regarded as at rest with regard to a hypothetical ether of space, while another, such as the earth, was regarded as moving relatively to the first and, therefore, also relatively to the ether. That is to say, the negative result of the Michelson experiments, before the special theory clarified the matter, indicated that to the earth's motion against the ether were to be assigned more complicated laws which were supposed to hold relative to the earth, that is to say, laws non-transformable by classic methods into those determined from the sun. The Professor, then, after stating that the Michelson experiment "appears as though it must be decisive," derives from it what appears to be a non-sequitur, though permissible, of course, as an hypothesis, that there is no such thing as a unique co-ordinate system with unique or non-transformable laws, and that "there can be no ether drift, nor any experiment with which to demonstrate it." He means that we may ignore the ether and assume that neither it nor any body within it is in a state of absolute rest and hence a unique co-ordinate system from which to determine motion; that there is, therefore, no actual, but only relative motion. That meaning is shown by this immediately following metaphysical language:

> "Here the contraction of moving bodies follows from the two fundamental principles of the theory without the introduction of particular hypotheses; and as the prime factor involved in this contraction we find, *not the motion in itself, to which we cannot attach any meaning, but the motion with respect to the body of reference chosen in the particular case in point.*"

We remark, in passing, the use of the words "two fundamental principles" to which we shall presently return, and the introduction of the Michelson experiment as one of the experimental proofs of the special theory, surely not convincing evidence, if, as we hope presently to demonstrate, the experiment must be regarded as the basis of the theory.

Passing this, however, the point to be emphasized is that, on our initial assumption of the Professor's meaning, the result of the Michelson experiment has no logical connection with the matter above quoted from Section V in relation to the second proof of the restricted principle. According to that principle, and assuming, as Einstein does, the adequacy of the Michelson experiment, a difference between the velocities of the ray of light in that experiment, accordingly as sent in one direction or the other, must have become evident. The failure of this difference, therefore, to develop, is, again on our initial assumption, not a proof, but a disproof of the restricted principle. This, Professor Einstein recognized when he intimated that the disproof is apparent only and that the seeming exception to the laws of nature disappears under the harmonizing light of the special theory. If the velocity of light appears the same to an observer on the moving earth whether the light move with or against the earth's motion, the restricted principle is annihilated, so far as concerns that fact, until restored by the special theory, so that that strange result of the Michelson experiments is an argument against and not for the restricted principle.

It must by now be fairly apparent that the inconsistencies and confusions noted are due to the fact that the Professor, having the proposition clear in his own mind, mistakenly considered that he had made clear in his book that the Michelson result is in harmony with the restricted principle, *assuming* the latter's extension into the special theory by the addition of the Lorentz transformations. Those transformations imply, of course, the attendant non-classic views of time and space. In the new space-time, motion ceases to wear its familiar aspect. The Professor meant to argue that motion is relative only and has no actual existence. The constancy of the velocity of light, in the primary sense, becomes true, therefore, also in the secondary sense, since otherwise, as we shall presently see, the motion of the observer would be actual. If there is no motion in space and in time, but merely an as yet inconceivable activity in space-time, the classic transformations can no longer be used. As all this is deemed by the Professor to have been deduced from the relativity of motion, the restricted principle must be deemed to include these newly discovered laws of nature, the only problem being to discover the apposite system of transformation, that is, the Lorentz equations.

These views of the Professor's meaning are confirmed by what we have already said and quoted and by several hints rather than statements made by him in leading up to the restricted principle, which evince non-classic notions of space and time inconsistent with the restricted principle as we have heretofore assumed it and which we have hitherto deemed to have been added to that principle in consequence of the Michelson experiment.

For example, after his statement of the problem of the stone dropped from the train, and his explanation that the stone travels in a perpendicular line to the train observer while in a forward parabolic curve to the embankment observer, Professor Einstein says:

> "With the aid of this example it is clearly seen that there is no such thing as an independently existing trajectory (lit. 'path-curve'), but only a trajectory relative to a particular body of reference."

It has been heretofore assumed that this statement that there is no actual path of the stone was a mere slip in phraseology, but it is now evident that it was meant as phrased. The meaning is that there is not in the universe any body in absolute rest, so that absolute motion may be determined or predicated. Now, it may be that there is no stationary ether with reference to which all the masses in the universe are in motion, as Einstein assumes, and it may be true, and probably is, that there is not, throughout the vast reaches of space, any body in absolute rest, although, of course, the truth or falsity of such a proposition is incapable of demonstration in our present state of knowledge. Nevertheless, by the very nature of our thinking processes, it has been hitherto deemed essential for us to posit or conceive such a state of absolute rest.

The stone, to the observer on the train, falls in a perpendicular straight line; to an observer on the embankment, it travels forward in a parabolic curve for a few feet; to an observer on the sun, to whom the trifling motion of the stone is as nothing, and the motion of the earth on its axis and in its vast orbit about the sun will be everything, the stone, during its brief fall, will traverse a space in miles in a different direction and in a different sweep of curve. To an observer outside of the solar system, and with respect to whom the solar system itself may be traveling at vastly greater velocities and in a still different direction, the path of the stone will appear to be again vaster in length and velocity and different in curve and direction.

In this connection, it may be remarked, incidentally, that even if the Michelson experiments had detected a motion of the earth in relation to the ether, that would not necessarily have been a proof of actual, as distinguished from relative motion. The reason is that the ether embracing our whole universe may itself be in motion, together with all its vast contents, in relation to another universe or other universes far beyond our own and separated from it and from each other by vast stretches of empty space containing no ether, and, therefore, forever barring from our knowledge or perception the existence of the countless suns beyond. The earth in that aspect could have an infinitude of motions and the path of the stone of our illustration might be one of infinite relativity.

Nevertheless, disregarding the last suggestion as immaterial to the present argument, the mind cannot avoid at least the conception that somewhere in space there could be an absolutely stationary observer, who would see the truth of the stone's path curve. This is a necessary result of the laws of thought, built upon race experience through æons of ages. It is true that it is based on that experience only and may not be valid for matters transcending experience. The contrary, however, seems, to man's mind as now constituted, inconceivable. This matter of conceivability will be treated separately in Chapter IX.

Again to repeat what we have already said so often, our knowledge of motion, as of matter, space and time is relative, expressing the form of mental reaction to modes of being, which, in their essence, escape us. The skeptic may assert the non-existence of all but the picture in the perceiving mind. Science has been founded on faith in the existence and in the persistence of the external realities underlying the mental appearances and in the persistence of their relations, as of position in time and space, antecedent and subsequent, cause and effect. This faith has resulted in the discovery of laws the truth of which, sufficiently for the purposes of human life, has been demonstrated by experience and successful prediction. Nevertheless, Professor Einstein states, when discussing the motion of the stone:

> "What is meant here by motion 'in space'? . . . In the first place we entirely shun the vague word 'space' of

which, we must honestly acknowledge, we cannot form the slightest conception and we replace it by 'motion relative to a practically rigid body of reference.'"

But science has had faith in the existence of space as mentally posited, and philosophy has appeared to establish that space and time, being the necessary conditions of thought, can never be known to us except as mentally posited. Therefore, an argument that "shuns" the word "space" is, according to classic science, not dealing with physical realities, but with metaphysical abstractions. But perhaps classic science needs correction. Perhaps we shall be able to pierce to a phenomenal ultimate still more profound than space, time and motion. Perhaps, even physically speaking, not metaphysically, motion, space and time, as we think them, are not ultimate actualities. Perhaps, as Einstein says, motion has no actual existence. Our poor faculties, if further developed, may be able to perceive motion of an object in time and space to be really something quite different from its present appearance.

But one thing appears to the writer reasonably certain. Although that view of motion is deducible from the secondary law of light and the special theory, it and they are not deducible, as the Professor seems tacitly to imply, from admitted premises, but this entire conception exists only as an hypothesis suggested by the Michelson result. It is the offspring and not the parent of that result. We shall now undertake to establish this contention. The two admitted premises, that is, the Professor's "two fundamental principles" before noted, are:

(1) No one can detect his own actual, non-accelerated motion, but only his motion relative to some other body.

(2) The velocity of light in a vacuum is constant whatever the velocity of its source.

No one is likely to deny the truth of either of these postulates; but we shall presently see an unwarranted extension of the meaning of the first in the argument which derives from the two the Einsteinian conclusion. Einstein himself treats that conclusion as self-evident, since he nowhere undertakes to demonstrate it for the benefit of his readers.

The first postulate, within proper limits, is axiomatic. Certain investigations appear to have established the second; and, indeed, it follows from the undulatory theory of light. If the corpuscular theory were true, each corpuscle propelled from a luminous body by a force which, acting alone, would give it a velocity of 100,000 miles per second, would have not only that velocity, but also that of the emitting body. If that body had a velocity of 86,000 miles a second in a direction opposite to that of the propulsion, the corpuscles, that is, the light, would have a velocity of 14,000 miles a second. If the velocity of the propelling body were in the direction of the propulsion, the light would have a velocity of 186,000 miles a second. On the undulatory theory, however, the light consists of the wave motion derived by the ether from the atomic motions of the propelling body, and hence its velocity depends upon those motions and not upon that of the luminous mass. Differences in the force of those motions would presumably show themselves in the number and lengths of the waves and not in their velocity. We may assume then the truth and physical naturalness of the second postulate.

The reasoning must be that, if light always travels at the same velocity (postulate 2), it must so appear to travel whatever the motion of the observer, since that motion cannot be detected (postulate 1). But postulate 1, regarded as an axiom, only states, or should only state, that motion of a body cannot be detected except in relation to some other body, that it cannot be detected in empty space, in the absence of bodies. It does not contain the proposition that this motion cannot be detected in relation to ether. That is not axiomatic, but the assumed result of the Michelson experiment. If such a motion could be detected, then the conclusion could not follow that light has the same apparent velocity to all observers whatever their respective states of motion may be. The observer will have a motion in relation to the ether and the light in the same way as in relation to any body or mass. Without the Michelson experiment, there is no reason to suppose that ether differs in this respect from any mass. We know little of the ether and cannot say whether it is a form of matter or not. Motion in ether, however, is a form of energy, and Einstein deduces that mass is energy. At all events, his own later conclusion is that light has mass.

Perhaps the Einstein position might be rendered clearer if stated in the following way: If one could detect the motion of his own body of reference, relative to ether, the medium of light, he would be detecting actual motion, since the ether is assumed to be at rest; but this detection of actual motion is in conflict with the first postulate. Thus there can be no detection of motion, actual or relative, of the body, relative to light; and, in that case all observers, however moving, must always find the same velocity of light.

The first postulate, however, is only axiomatic in so far as it is confined to the meaning that the motion of a body has never been detected in relation to space or otherwise than in relation to *matter*. That statement, common and direct experience supports. But no statement may be made, as a matter of common or direct experience, of motion in relation to ether. The postulate, therefore, cannot include such motion; and no reasoning based on its axiomatic truth as so extended is valid. The only way to enlarge the postulate to include the ether is by way of the Michelson experiment.

On the other hand, let us, as an *hypothesis,* add to the first postulate, that an observer cannot detect his own motion relative to the ether, or even that there is no ether, and that actual motion in space cannot be detected. If light's velocity should appear to vary according to the observer's motion, he would have detected his motion in the ether, or, if there be no ether, his actual motion in space, which is contrary to the hypothesis. But the inability to *detect* such motion and consequently differences in the relative velocity of light may be due to unknown natural causes; it is not a logical proof of the non-existence of phenomenal or physical motion. Certainly nothing in physics, except the Michelson experiment, suggests such an hypothesis.

We may reach the same result in other ways. Let us return to our old friends, the train and the embankment, and let us suppose, at the tail of the train, a flat car with its rear, at the instant of the beginning of our observation, coincident with a certain place on the embankment, which we may term the origin. Now as the train moves let us suppose that at this instant the raven of our previous illustrations, or, better still, a lifeless projectile, be propelled perpendicularly upward from the floor of the flat car at the origin point. To an observer on the embankment at the origin, as we have also already noted, the projectile will seem to be traveling on a forward slant, while to the observer on the car it will seem to be traveling vertically upward. If, on the other hand, the projectile be similarly propelled from the point of origin on the embankment, it will seem to the observer on the car to be moving backward at a slant, while to the embankment observer it will seem to be moving vertically upward. The reciprocal appearance to both observers, therefore, will be the same though their explanations will be different.

In the first case, the embankment observer will say that he himself is stationary, that both the car and the projectile are moving and that the reason the car observer sees the projectile as moving vertically upward is that as it moves on a slant forward the car observer is constantly traveling under it so that he sees it always above his head. The car observer, deeming himself at rest and the embankment as moving, will regard the projectile as moving perpendicularly only, but appearing to the other observer to be moving on a slant by reason of the latter's motion. The same explanations reversed will apply to the two points of view in the second case posited. The explanation, of course, is that the projectile has, or appears to have, two states of motion, the rest or translation of its system, and its own independent motion upward. Each observer is sure that the other's system is moving and the projectile propelled from it moving with it as well as upward, the slant being the resultant of the two motion-producing forces;

and that the projectile on his own system has but one motion, that of the upward propulsion. Phenomenally or physically speaking it is, of course, true that every body has the motion of its own system besides any independent motion that may be given it relative to the system.

Let us now, however, substitute for the projectile a beam of light, not forgetting that light has its own motion only and in its own medium. In other words, it is independent of the motion of its source. To the embankment observer, therefore, the beam of light propelled vertically upward from the car will be left behind by the latter's motion and will appear to ascend vertically from the embankment origin. According to classic notions, however, it must appear to the car observer, according to the other's calculation, to be traveling backward at a slant, in the same way as though it were propelled vertically upward from the embankment at the origin point. In the latter case, the embankment being regarded as at rest, the motion of the beam will appear vertically upward to the embankment observer, who will again figure it on a backward slant to the train observer. If, however, the embankment be considered in motion, as the train observer will consider it, the beam ought again to be left behind and to appear to travel vertically upward to the train observer and, by his calculation, on a forward slant by the embankment observer.

In these cases, we see that our two observers can no longer exchange explanations that will be satisfying. By reason of the fact that the motion of the light is independent of that of the system which emits it and in a direction perpendicular to that of the observers' relative motion, appearances are created, irreconcilable from the different viewpoints. Each observer, regarding his own system as at rest, must expect the beam whether propelled upward from his own system or left behind by the other, to appear to him always traveling perpendicularly and never on a slant. Now, of course, the train-embankment illustration is imaginary. The matter cannot be tested by that crude method. There are apparently only two ways in which to establish the truth. One is by the testimony of an hypothetic observer indubitably in absolute rest (and we know of none such), the other, by adequate experiment. None such, however, has been devised unless it be the Michelson experiment.

It is a curious reflection that in these illustrations, it is of course impossible to observe the course of the light's travel except relatively to non-ethereal matter, since the course of light can only reveal itself by reflection of the waves by encountered particles of such matter.

The above difficulties are created by the relativity of motion, the fact that we cannot determine motion of one body except in relation to another, that we cannot be sure where, if anywhere, actual motion exists. The observer on the imaginary outside system in actual rest alone could perceive whether the train was at rest or the embankment or the precise nature of the motion of each, if both were actually moving. There is but one possible explanation, according to classic standards, of the fact, if it be a fact, that the beam would travel vertically to both train and embankment observer. That is that neither the train nor the embankment has motion relative to the light or its ethereal medium or empty space; not merely that such motion cannot be detected, but that it does not exist, unless we assume Nature to be guilty of some trick of legerdemain. But the lack of motion or even of motion detectable in relation to light is far from being axiomatic. Both it and the supposed fact it explains are assumptions without scientific support or suggestion other than the Michelson experiment.

However, whether the Professor's views be regarded as based on the relativity of motion or as constituting an hypothesis suggested by the Michelson experiment, we trust they are now reasonably clear. According to them, there is not in the universe any body in actual rest. Neither is there any body in actual motion. Motion is purely relative. Although body A may be at rest relative to body B, which from A appears to be moving, to an observer on B it will be A which appears to move, and neither is actually moving. Furthermore, the Professor denies or at least ignores the existence of an ether in space stationary as to moving bodies. He insists that no unique co-ordinate system, therefore, may be conceived to be embedded in it, from which to admeasure actual motion. Phenomenal motion in space and time is an illusion. From these premises he deduces an underlying phenomenal reality hitherto unsuspected and which, if not noumenal, suggests, as we shall see, the hypothesis of an infinite series of still profounder phenomena. The supposed realities of the universe become relativities only.

This is pragmatism with a vengeance. The mathematician is satisfied if these relativities are worked out for him in the form of mathematical abstractions. If they are, he assumes their truth irrespective of the power of the mind to picture, symbolize or conceive them.

At this point it becomes important to attempt in some way to visualize what Einstein means by the non-existence of actual motion. If the reader were to take a train at the Pennsylvania station in New York for a trip to Washington, however perfect the uniformity of the train's motion might be, he, of course, knows, from the common-sense standpoint, that it is he on the train who is moving and not the various stationary objects which he passes and which appear to be moving in the opposite direction. He knows that something has been added to the train, which makes it move, that something being the power, supplied by the locomotive furnace and boiler, of the tremendous molecular action engendered by them and impressed upon the piston in the form of steam. Einstein also knows this. What can he therefore mean when he implies that the train is *actually* not moving any more than the embankment? He must mean that the force developed on the train, even as the force inherent in the body of a gravitational field, does something to the adjacent space or space-time, which, from the point of view of the observer, creates the *appearance* of motion, or rather makes for itself a space-time of that character.

Motion in itself is, both philosophically and scientifically, incomprehensible to us. In some way, impenetrable to finite intelligence, that intelligence reacts in the form of motion in space and time to the phenomenon known as change of position. The energy on the train, then, operates on the adjacent medium in such a way, or creates one of such a character, as to elicit for us the appearance of a motion which is not actual. The train is not really moving. We do not need to be convinced that the earth itself as a whole is not moving backward from the train, or that not even the embankment is moving back beneath the wheels. What is happening, according to Einstein, must, therefore, be that, at each operation of the force of the piston, something, presently inconceivable and mystic, is happening to space or space-time, twisting, contorting or doing something else to it or our view of it, which creates mentally the appearance of motion, so that neither the train nor the embankment is actually moving, but each only relatively.

If one of them be in actual rest, the theory as formulated collapses, because, while to an observer on the other, it might, by an optical illusion, appear to be moving, being "actually" at rest, the direct cause of the apparent motion must be in such other body, which must then be in "real" or "physical" motion. It is only if the disturbance be in what we call the intervening space that both bodies would alike be subject to space-time distortion, as required by the theory, because due to a common cause extrinsic to both, though arising from an energy disturbance present in one of them. The train motion is not a motion in space during time but an unpicturable activity in space-time.

It is proper to state at this point that, although we have had and shall have something to say on this matter of space-time twist or curvature or distortion or warp, Einstein himself trusts the intelligence of his reader to derive some such conclusion. As we have before remarked, he avoids the construction of pictures of the new world necessitated by his revolutionizing views. Such a twist is the necessary result of what he does say, but is not presented by him further than in the mathematical formulation of his space-time world, in the statement, which we shall presently encounter in his discussion of the extension of the special into the general theory, that in gravitational fields space ceases to be Euclidean, and in the mention of a curved space in relation to his new conception of the universe as a whole. In an appendix he also refers to light deflection as being due to space curvature.

From what has been said it must be apparent that for Einstein the problem is basic, to determine whether mere relativity of motion, as distinguished from actual physical motion, is as true in the case of accelerated as in that of uniform, rectilinear motion. To discuss this, we must anticipate somewhat the general theory, the development of which begins with the application of the principle of the relativity of motion to accelerated as well as to uniform, rectilinear motion. The Professor having called attention to the fact that systems of uniform, rectilinear motion are such only as viewed from systems of the same kind of motion, and not from systems having accelerated or rotary motions, insists that the mind cannot be satisfied until it reaches some reason for this difference. Why, to an observer on the earth, should the stars appear to travel around it? This question can hardly require an answer unless one already assume the truth of the Einstein theory, including the non-actuality of motion, the non-actuality of the earth's axial rotation. But the question is intended to evoke the theory as the answer to it. The Professor deems the solution to lie in a physics conformable to his general theory, for the reason that the equations under that theory hold for every body of reference, whatever may be its state of motion. In other words, those equations disclose the reality, independent of the differences inherent in the relative points of view.

Many will not be able to perceive why motion, either uniform or accelerated, cannot be deemed "real," even though its reciprocal appearance from the moving systems be relative, why the appearance of motion in the embankment of our frequently used illustration, as viewed from the train, or the apparent daily motion of the stars about the earth, may not alike be deemed illusions only, and not "realities." Certainly in the latter case there ought to be actual motion somewhere. No scientist is likely to believe there is such a causal connection between such distant objects, as, without real motion in either earth or star, will produce the reciprocal appearance of motion in each. Motions may be real in a phenomenal way, notwithstanding that, even phenomenally, they are not as we perceive them. The corresponding activity may be real and present. In fact, the "reality" of the motion of rotating bodies is supposed to have been empirically demonstrated. The equatorial bulge of the sun and planets indicates it, and certain famous pendulum experiments have made the earth's rotation all but visible. This branch of the subject will be better understood when we reach the general theory.

The Einstein position is that, because motion, uniform or accelerated, has no actuality, the appearances must be due to some condition of their medium. The real things in nature are energies, activities. They manifest themselves to consciousness in forms differing according to the point of view. The space form is not real; neither is the time form. The real form of the medium containing these activities is an as yet inconceivable mixture of space and time created by them. And this leads us to a more detailed consideration of a theoretic explanation of the results of the special theory.

CHAPTER VIII

THE RESULTS OF THE SPECIAL THEORY MORE CLOSELY EXAMINED. THE FOUR DIMENSIONAL WORLD

BEFORE attempting a picture of the results of the special theory, it will be useful, first, briefly to summarize them. When two bodies move in relation to each other, if the observer is on the one, it is the other that is moving, while, if he is on the other, it is the first that is moving. The time units will be longer and lengths will be shorter and mass will be greater on the relatively moving body as viewed from the relatively stationary one; and precisely the same results will be obtained *vice versa*. Nevertheless, the differences are actual. Each body has its own length and time measurements and mass according to some condition of energy within it which corresponds to its state of motion. The motion, however, is not absolute or actual, but only relative to that of some other body. The shortening of lengths on the relatively moving body occurs in the direction of travel only. Across the direction of travel, there is no such effect, so that if your system be traveling rapidly enough, all bodies will be flattened in the direction of travel, but will recover again, if the shortened length be pointed perpendicularly to its previous direction.

But these changes cannot be detected by an observer traveling with them. His measuring rods, including the retinas of his eyes, will suffer corresponding changes. By no device, accidental or experimental, according to the mathematicians, can these changes be discovered. For all we know the earth, in the multitude of its motions as a part of the moving solar system, may be traveling, relative to some unknown body, at some velocity approaching that of light, say at 161,000 miles per second, a velocity which would double the length of its time units and halve the lengths of all bodies on it, in the direction of travel, as compared with what they would be if the earth were relatively at rest. Turn these bodies at right angles and their length will immediately double. We could not perceive it, however, any more than we could perceive our motion, whatever the velocity. To us everything on earth would be the same whether we were traveling at 161,000 or 18 miles per second.

If two human beings were alone in space, moving toward each other at uniform velocity, neither would be conscious of his own motion, but each would conceive of the other as moving toward him. When they came into violent collision, however, they would certainly, each, be conscious that a force existed somewhere, whether in one or the other or both of their bodies, or in the intervening medium. In the philosophic and scientific sense, the motion of these two bodies, in the form of our relative knowledge of it, need have no existence, but its underlying reality, though unknowable to us, exists. In the case supposed, since, according to Einstein, it exists in neither body in the form of actual motion, but its effects are, nevertheless, actually to change time and measurement units and mass on both, and to make both move for an outside, relatively stationary observer, the only apparent solution is that the motion-producing cause *lies in the strain or twist or curvature of the intervening space produced by the bodies or by that energy in them which expresses itself to us in the motion thus indirectly caused by it.*

Now space, like time, like sensation, is a mental form, a condition of consciousness. Mind cannot perceive or know that which is the condition of its functioning, even though it express a noumenal reality. Speculation, therefore, as to the inherent nature of space has hitherto been deemed non-scientific. Here is apparently an attempt to pierce the veil, not only by positing space or the substituted medium, space-time, in its essence, but apparently attributing to it as a quality, susceptibility to curvature, a quality or mode, moreover, not only admittedly inconceivable, but inconsistent with the space form which is the universal condition of all present conception of an external world. But perhaps a better interpretation of the Einstein meaning is that, as we only know space and time in connection with bodies, their extension and motion, the confusion arises from the mental separation and abstraction of space and time. They do not exist apart from the totality of phenomena and their action. Their appearance to us is due to our special condition. With different faculties and when dealing with high energies, we should be able to perceive a new space-time reality, not noumenal, to which we are able, even now, to pierce by logical processes, and which we are able to demonstrate by empiric proof.

For purposes of further clarification, let us now consider certain problems. Let us first suppose two light signals flashed at a considerable distance from each other at the same instant, from the point of view of a stationary observer, on the embankment of our frequent illustration, midway between the flashes, and then let us suppose the train to be moving at the velocity of 161,000 miles per second, so that the train observer's position at the instant the flashes are emitted, is at the midway point. Will these two flashes reach him at the same time?

Einstein's answer is that they will not. He will see the flash toward which he is traveling before the one from which he is traveling. The first will reach him before he is overtaken by the second. But, as we have before noted, the Professor intended this answer to apply only to classic standards. According to the basic principle of the theory and of the Lorentz equations, the flashes would still be simultaneous, as otherwise the train observer would be able to detect his own motion through the ether. At all events, according to the theory, each observer, provided with the proper instruments, will measure the velocity of both flashes at precisely the same amount of 186,000 miles per second. The embankment observer will decide that they are simultaneous, but that they cannot appear so to the train observer, who, unaware of his own motion, is rushing into one, from the former's standpoint, at the velocity of 347,000 miles per second and away from the other, which is, therefore, overtaking him at the velocity of only 25,000 miles per second. After this experiment, however, if the two observers should meet and the embankment observer should inform the other that the reason he considered successive what were in fact simultaneous flashes was because of his motion, or rather, because he thought himself at rest and the rear flash to be traveling toward him at 25,000 miles per second and the forward flash at 347,000 miles per second, the train observer must reply that this cannot possibly be true, as he measured the velocity of both flashes by unquestionable methods and obtained the same unquestionable result in each case, a velocity of 186,000 miles per second. The difficulty is that each observer assumes, without right, that his own system is in absolute rest and the other in absolute motion.

These considerations lead to another problem, the answer to which supplies us with surprising results, which make clear the necessity of some kind of scientific theory or picture to account for the mathematical reactions. Let us suppose that we have three observers whom we will designate as *O, O′*, and *O″*. *O* is stationary. *O′* is on a system some distance away from *O*'s system and which is moving, relatively to it, to the right, at a velocity of 80,500 miles per second. *O*'s system is between that of *O′* and that of *O″*, which is moving to the left at a velocity of 80,500 miles per second. The relative velocity of *O′* and *O″* will, therefore, be 161,000 miles per second. *O′* will see a yard stick on the *O″* system shrink to one-half a yard, while *O″* will see a yard stick on the *O′* system incur precisely the same shrinkage. But relatively, to *O,* both *O′* and *O″* are traveling at the velocity of 80,500 miles per second and *O* will see the yard stick on both the *O′* system and *O″* system shrink only to nine-tenths of a yard; and similarly both *O′* and *O″* will see the yard stick on the *O* system shrink to nine-tenths of a yard.

Now the attempt was made above to save the special theory from the *reductio ad absur-*

dum involved, from ordinary points of view, by reason of the same shrinkage occurring on each of two relatively moving systems, by the suggestion that the motion energy expressed itself in an actual space-time warp in some way covering both systems, and, therefore, actually shortening the dimensions of both systems, though not appreciably to either observer with reference to his own, owing to the corresponding shortening of his measuring rods. This would not be a satisfactory explanation, however, since it would not account for the fact that the changes on one system would be perceptible from the other. We might think this should not be the case, since the measuring appliances on both systems are similarly affected. The relativist mathematicians, however, insist that from one system the changes in the other must be discernible, as otherwise motion relative to the ether could be detected; and we must accept this conclusion.

The difficulty is increased, however, by the solution of the last problem to the effect that while, in the case of the two systems moving in opposite directions and therefore at relatively double speed, the shortening is the same quantity on each, to the relatively stationary observer between them, the same supposed actual space-time warp produces a much smaller shortening. This negatives the idea of an actual warp which includes all the systems, since the effects are thus different. Moreover, it could not plausibly be imagined that if one moving system were a vast space-time distance from a relatively stationary one, the supposed warp produced by the motion energy of one system should, in all cases, extend through the intervening distance.

Perhaps the *reductio ad absurdum* may still be saved by supposing that there may be "actual" motion, which warps the space-time of its own system only. This being so, the observer on the actually moving system perceives the phenomena of a stationary system through the warp of his own space-time only. The stationary observer perceives the system through its own warp. The moving observer sees the phenomena of another system with an actual and opposite motion through both warps, thus increasing the effect, while if both systems be actually moving in the same direction and velocity relatively to the stationary one, the two warps neutralize each other. This explanation, however, assumes actual motion, which Einstein denies, and contains the same relativist proposition as before, that the changes on one system are detectible from the other.

On reflection, however, it will be seen that in this discussion we have been guilty of an egregious fallacy. True, we have been using the term "space-time," but it will be clear from the context that we have been thinking of it in precisely the same way as we think of space. We have been writing as though we know something on a subject of which we know nothing. We have been applying the ideas of our three-dimensional world to a four-dimensional world, which, entirely inconceivable, is only made manifest in the abstractions of mathematicians. We have forgotten that, as an eminent mathematician (Bertrand Russell) has said, "mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true." The mathematician is satisfied with consistent results from assumed premises, and leaves it to others to attempt to conceive or picture them.

The relativist wants us to understand that the real world is one of four dimensions, of which time is one as truly as any of the others.

That world he views as, in a sense, absolute, though it can only be known relatively by observers having varying relations to it. The forms in which it and phenomena occurring in it appear to observers depend upon their position and motion and those of their respective measuring appliances. In other words, the form of the appearance of the real world and its phenomena depends upon the point of view. This must be the real meaning of relativity.

It is not proper in relation to such a world to talk of measuring lengths and time units. It contains no such things. They constitute the reaction of particular observers to its real phenomena. According to their motions, the time unit lengthens and the measuring rod shortens actually in one sense, but not in another. The measuring rod will contain the same number of molecules, whatever may be the point of view. A time unit as such simply does not exist. The appearance to us of these measuring values is as we separate them out in accordance with our own points of view. Length and duration are not inherent in the external world. They constitute our reaction to external realities. Our own conditions, however, such, for example, as our state of motion, give different mental meanings to them. In a sense, we create our universe. We supply differing scales for the measurement of the same realities, which, for the same observer, change precisely as the scales change. Each observer, according to his own condition, is interpreting in his own way the absolute facts of the space-time world, and the appearance will vary according to the condition, particularly according to the state of motion. Compress the whole universe to the size of an orange, everything in proportion, and no one would be the wiser. Perhaps, in fact, that would be no change at all.

In our own condition, *our minds have created space and time.* They are the mental reactions of our particular condition to phenomena occurring in the real world. In that world, a straight line ceases to be the shortest distance between two points. In fact, it is improper to speak of "distance" at all, unless we differentiate it by a special name such as "space-time distance" or "interval." The longest "world-line" or "interval" between two events, might include the shortest space distance. The space-time curvatures and twists and the consequent shortenings and lengthenings exist, but are effected for observers by their own different motions and conditions, these in turn not being actual but relative.

Thus, the only possible explanation of the results of the special theory is no satisfying explanation at all, for we are called upon to posit, as containing the phenomena of nature, an admittedly inconceivable, perhaps noumenal, continuum, which certainly could have no scientific interest, were it not that its propounder believes that, inconceivable as it is, it is yet able to penetrate to our mentalities through empiric proofs. This explanation is a non-picturable world of four dimensions.

This notion of a kind of absolute four-dimensional world is, from every aspect, the only logical outcome of the Einstein assumptions. On the basis that light's velocity is a constant *stated quantity,* irrespective of the motion of the observer, the Lorentz transformations were produced. The results of those transformations seem remarkable unless that basis is constantly borne in mind. By their application it has been seen that a velocity of 186,000 miles a second for light in relation to a stationary system remains the same for a system, in relation to which, according to classic standards, it should be 25,000 miles per second, or even 347,000 miles, according to the direction of the ray. This is due to the changing values of the time and space measurement units, automatically yielding compensatory results by reason of the basic assumption. But what reason can there be to assume that the never failing outcome of a velocity of 186,000 miles in one second for light in all the differing cases, is a constant, when it is measured in mile units and second units differing in each of the supposed cases? Is a velocity of 50,000 miles of a certain kind in 50/186 of a second of a certain kind, although it figures out 186,000 of such miles in one of such seconds, the same as a velocity of 186,000 of a different kind of miles in a different kind of second? From this point of view, the Lorentz transformations alone do not solve the problem, and there is no reason why our wizards of Chapter VI should have emerged from their tubes at the same moment.

But on the assumption of the space-time world, this difficulty disappears. The measuring values and that which they measure do not really change. They are always the same. The points of view of the measuring observers, their mental scales, change. The inherent energies represented by the same numerical velocities of the different scales are the same. Number is absolute; scale is relative. True, there is not in the four-dimensional world a real measuring length or time unit. The facts of that world are of an inconceivably different kind, but, separated out by the mentalities of observers of different viewpoints, these standards *appear* as real things. That for which the appearance stands is invariable, but to observers of different relative motions the appearances differ. The mile stick and the second are always really the same so far as they stand for something in the real world, but actually change in magnitude together with what they measure, according to the state of the observer.

The changes then are, after all, illusions, but not in the ordinary sense. The illusions have fixed and consistent mathematical relations. They are supposed to constitute our interpretation of deeper lying physical realities, in the same way as the philosopher supposes all phenomena to constitute our interpretation of noumenal realities. In the real world, space and time and measurement units thereof do not exist separately as we conceive them. Things never move in space directions alone. They may move in the time direction alone, when they merely endure but do not travel. If they travel, their direction must be a composite of space and time. Our experience is too limited to enable us to perceive these real things. It makes us react to them in terms of space and time as separate things, and of measuring rods and clocks as standards for them. The new mathematics has pierced through to them and indicated, if it cannot picture, the real four-dimensional world in which they exist.

Professor Einstein does not attempt to picture or explain this world. He is content to accept Minkowski's mathematical formulation of it. The conception of such a world is not new in mathematics or philosophy. Space, regarded as the form in which the mind reacts to phenomena of motion, is conceived by us only in terms of three dimensions. This has not blinded thinking men, however, in their metaphysical and mathematical speculations, to the possibility of an infinite number of dimensions, as well as of powers, to speak in finitely comprehensible terms, in the unknowable Absolute, of which all phenomena in their last analysis are manifestations.

Neither is the view of time as a fourth dimension a new thought. An analysis of our notions of time and space shows a certain identity or at least close relationship between the two. The mind conceives of substantive existence, in the last analysis, in terms of force. The static existence of this force, if the expression may be allowed, manifests itself in the concept of extended matter or substance. Its kinetic or motional aspect manifests itself in those mental strains which form the complex of motion of matter in space and time. A human being alone in space except for a distant moving body, would form his concept of its motion through the strain in the ocular or other muscles in following the flight, and the association of the visual with former tactual appearances, and the like, already sufficiently analyzed. His mind can conceive of this motion as forward, backward, or in any direction, and hence of space as extending in all directions.

If now a series of stationary posts are placed in space in a straight line, along the positions of which the moving body travels, the intuitional concept of time arises from the succession of events realized by its progress forward from station to station. That concept, however, is not applicable in the same way to a reverse motion of the body. This is due to the fact that in the finite universe of conflicting forces and never-ending movement, progress in any direction involves frictions, wearing out, change, death. In an infinite universe of non-complex and hence indestructible beings, time becomes converted into eternity, disappears as a necessary concept. This would certainly be true if our observer, the moving object and the posts were alike incorruptible and changeless. In such a case time would have no meaning. Its reality is in the eternal restlessness of things. Our observer's mind, however, functions, with finite memory, in a finite body, which develops and wears out and dies, so that for him, observing the retrograde motion of the traveling and corruptible object, as it reaches, in succession, the same station posts previously passed in reverse order, those posts are seen in comparison with their images recalled in memory, and with such changes, greater or less, as the interval will have brought in the moving body, the station posts, and the observer's mortal self. Thus for us the time concept, though so closely related to that of space, can have a forward direction only. For mortals, that which has happened, can never happen again, except in reminiscence, and then it is a different happening. "The moving finger writes, and having writ, moves on." Four-dimensionalism may involve a different view. For it, all events, past, present and future, may be coincidentally existent, developed for the individual as the space-time world-line of his travel encounters them.

Time, then, is conceived and measured by us in terms of motion through space. The day is, so to say, the concept of the repeated motion of the sun about the earth, or, in more precise terms, the concept of the sequential motion of the earth about its axis. Similarly, the year is the concept of the sequence of the earth's motion in its orbit around the sun. Such time measurements are essential for finite beings living in a changing world of finite parts.

In such a world an object, in order to be perceptible to consciousness, must *endure.* The mind cannot perceive it until the eye, by the movement of its muscles or by the associations of perspective with other muscular movements, has traveled over it, taking in by complexes, its extension in three directions, or until the hand or other member has touched it, traveling over its surfaces. Any such sequences of motion take time in space, and, as the perception is impossible without the motion, the object cannot exist for us in space without the time element.

Thus, each of the station posts of our illustration, though deemed at rest, may be said, in a sense, by its persistent endurance in space, by the quality of the static energy which constitutes its existence, to be moving along the time dimension, in the same way as, by the addition of kinetic energy, it could be given apparent motion in space. That, at all events, is the view of the four-dimensionalists; and in this sense the space medium may be said to have four dimensions. The energy of the enduring post constitutes its travel in time. From the viewpoint of the moving observer, who deems himself stationary, it may be said to be traveling in space also, that is in a composite space-time. If it helps anyone to think in that way, no harm is done; but it is a fair inquiry whether, even granted our qualified abilities of perception, there is any analogy between travel in space and endurance in time. The only readily perceptible relationship is the measurement of endurance in terms of space motion.

The serious meaning underlying the space-time conception is that we do not get a true idea of the position in the universe of an event when we are given space co-ordinates only. The complete picture can only be afforded by the addition of the time co-ordinate. To say that a battle took place in Paris is to say that it took place so many miles and in such a direction from New York, but the event is not truly placed until we add the date, that is, a dimension measured, in days or years, in a number of certain periodic motions, from another time, such as the birth of Christ. To determine the full time and space relationship between two objects in space, speaking in orthodox language, one must know their distance apart in terms of an agreed length unit, and if one was in its place before the other, the time between them in terms of a time unit based on some periodic occurrence. In the new language we are said to be living in a space-time world. The persistence of objects and the duration of events are viewed as travel along the time dimension in a sense analogous to that in which we seem to see objects moving along space dimensions. This conception, however, has never been realized by anybody except in mathematical symbols.

The non-Euclidean mathematician Minkowski developed it in that way, in connection with the special theory. We have already mentioned the non-Euclidean geometers who, for more than a hundred years, have been creating the geometries of imaginary worlds, where our axioms do not apply. Until now their efforts, tremendously ingenious though they are, have been quite generally viewed as so much intellectual power wasted in devotion to puzzles having no real relation to science. If Einstein be right, that view is mistaken. All these painstaking labors were, more or less consciously, a preparation for his epochal discovery. At all events, Einstein uses this already made instrument for his mathematical formulation of the space-time world. We shall now endeavor to give a most elementary idea of this formulation. Those who do not like mathematics may omit the remainder of this chapter, but it would be wise not to do so, as the treatment is elementary and easy, and will give some notion of the foundation upon which rests the space-time conception, and, together with it, the general theory and the new law of gravitation.

The following is a method of determining the distance or length of the line between two points in a plane, in reference to a Cartesian co-ordinate system, to neither axis of which the line is parallel. It makes no difference if this line be curved inasmuch as, for the purposes about to be described, the selected distance of the two points must be so minute that the length will be practically the same whether the line be straight or curved. We shall designate the minute distance in question as the distance element. From each end of the distance element are dropped two perpendiculars, one to the x axis of the co-ordinate system and the other to the y axis, extending the perpendiculars in such way as to form, by their respective intersections, together with the distance element, the sides of a right angle triangle, of which the element is the hypotenuse and the respective x and y co-ordinate differences of the two end points are the remaining sides. We then have a method of measuring the distance element by use of the Euclidean proposition, known as the theorem of Pythagoras, that the square of the hypotenuse is equal to the sum of the squares of the other two sides. Designating then the x co-ordinate difference as x and the y co-ordinate difference as y, we have the result that the square of the distance element will be equal to $x^2 + y^2$.

Let us now select different co-ordinate axes, for example, x' and y', and draw the same perpendiculars as before from the ends of the same distance element to these new axes, forming again, in the same way, a right angle triangle, of which the distance element is the hypotenuse. You may place your new co-ordinate system at any angle you please in relation to the first one and still, since the distance element is always the same, you will always have the same expression for it, though in terms of the new co-ordinate system, that is to say, the square of the distance element will now equal $x'^2 + y'^2$. Your new right angle triangle may be of an entirely different shape from the first one, but the same relationship must, of course, still hold.

If we now desire to place our distance element in three-dimensional space, that is to say, in relation to a Cartesian co-ordinate system of three dimensions, we shall merely, in order to obtain the quantitative relations, have to drop our perpendiculars not only to the x and y axes, but also to the z axis, but the relationship will still turn out the same; and we shall have the result that the distance element will be expressed by the form $x^2 + y^2 + z^2$ or on another co-ordinate system by the like form $x'^2 + y'^2 + z'^2$.

The distance element, thus, being always the same and expressible in like forms of equal value, whatever the stationary Cartesian co-ordinate system selected may be, is itself a quantity known as an invariant. As above shown, in two-dimensional space it is formulated in various expressions always involving two quantities; in three-dimensional space in various expressions, always involving three quantities. The question immediately suggests itself, whether the distance element is still an invariant, which may be similarly expressed in terms of the co-ordinates of a moving Cartesian system at any given time, by the Lorentz transformation. We have seen that, according to the special theory, the answer must apparently be in the negative. The motion, even at any particular instant, renders the time and space quantities variable, and introduces into the equations three new quantities, the lapsed time, the velocity of the system and that of light.

Now the strange variance of time and space quantities may be expressive of a world of four dimensions. The non-Euclidean geometry would, in that event, become useful. If a straight line may no longer be the shortest distance between two points, and parallel lines may now meet, strange results must follow. Minkowski, therefore, set out to discover whether he could devise, by the use of the Lorentz transformation, a formula for the distance element in the four-dimensional world, which would contain four terms and be analogous in form to that above set forth for the three-dimensional world, and invariant in relation to any system, in spite of the apparent disturbance in values caused by motion.

When he applied the Lorentz equations to this determination of the distance element, now become the "interval element" in the space-time world, the result was, at first, an equation which contained, besides the x, y, z quantities, the new quantities, v, the velocity of the system, c, the velocity of light, and t, the time. He did find, however, that when he took the form $x^2 + y^2 + z^2 - c^2t^2$, and transformed it by the Lorentz equations, the vs cancelled, and it turned out equal to the form $x'^2 + y'^2 + z'^2 - c'^2t'^2$.

Such is the fondness of this class of mathematicians for analogues and their trust in the strength of theories derived therefrom, that he and Professor Einstein have exhibited the greatest confidence that this equation expresses the truth of a four-dimensional world, the minus quantity, which is the departure from the analogy of the previous equations, expressing, in some subtle and incomprehensible way, the inconceivable twist or curvature of the fourth dimension, time.

In order to make the analogy complete, by rendering the form of the equation identical with the three-dimensional form, these mathematicians first concluded to adopt c, the velocity of light, as a unit with, therefore, the quantity 1, which immediately reduces the expression c^2t^2 to t^2. The minus sign before that expression, however, being a differentiation from the Pythagorean formula, they then adopted for the expression t, the imaginary expression $\sqrt{-1}t$, which immediately transforms the last quantity of the expression to $+ t^2$, thus giving, with these corrections, the following equation:

$$x^2 + y^2 + z^2 + t^2 = x'^2 + y'^2 + z'^2 + t'^2.$$

This merely means that, in adding to the expression the term containing the quantity *t,* we are adding, as expressive of the four-dimensional continuum, an inconceivable negative quantity. The above exposition does not cover details and is merely intended as an elementary guide.

It will be observed, then, that the attempt at mathematical expression for four-dimensional space involves the use of the great Pythagorean theorem, the backbone of Euclidean geometry, that the time quantity appearing in the equation finally produced is a negative and has no comprehensible value, that the reasoning is based largely on analogy, and that the resulting invariant interval element is a mere expression not represented by any conceivable reality. This interval element, of course, ceases to be distance. It is the expression for the interval, not between two points, but between two adjacent events in the space-time continuum; and its theoretic extension constitutes a world-line which, in the space-time world conception, means travel, in a united space and time, from event to event. A direct method of the enlargement of the minute, space-time interval element into a real world-line has, so far as the writer is informed, not been devised, except by the methods of classic mathematics. Only the element is conceived, by reason of its minuteness, and not very logically, to be susceptible of treatment in accordance with the Pythagorean proposition, but neither that proposition nor any other classic formula ought properly to be applicable to a four-dimensional world.

The manner in which these ideas are applied to the general theory of relativity, or, at least, to the new law of gravitation, will appear later.

CHAPTER IX

THE TEST OF CONCEIVABILITY

As one after the other of Einstein's ideas or those involved in his mathematical results have been developed, we have, perhaps too often, called attention to their inconceivability. Inconceivability means the inability to posit or picture in consciousness, either directly or symbolically; and it may connote such inability as inherent in the nature of mind, or in relation to the scientific material at present available or to the power of mind at the present stage of its evolution. If the inability be inherent, reasoning on the basis of the illegitimate concepts is impossible.

A man can form no conception of the magnitude of the earth in relation to his own size and standards of measurement, much less of such a magnitude as that of the solar system. Yet that system, adequately for his purposes, is conceived symbolically by a mental picture of so many little balls revolving about a luminous central ball in elliptical orbits, each attended by similarly revolving smaller satellites, and all the balls rotating also on their own axes. The mathematical relations are mentally added to the little picture, with the surprising result of predictions fulfilled. Thus, while the solar system can by no means be conceived in its actual magnitude (relative to our usual magnitudes) it can be adequately conceived in symbols of ordinary magnitudes.

No less thinkers than Mill and Huxley seem to have been misled into confusing difficult with impossible conceptions. The former has advanced as an example of a conception impossible in one age and possible in a more advanced one, that of people standing with their heads apparently downward on the part of the earth opposite to that on which the thinker stands. It is true that, until recent centuries, the rotundity of the earth was not a matter of general knowledge, and the conception named might have been believed, even by thinking men, impossible. Their belief, however, would not have made it so. It was merely difficult, because men were not accustomed to the idea. Given the new scientific material, the rotundity of the earth, and possibly also the idea of gravitation, there is nothing in the mental nature to prevent the picture. It is still a difficult one, but may be realized now in at least two ways.

In the first place, we might imagine a person walking half way around the earth, visualizing his travel on large sections of it at a time with a very gradual curvature. In this way, the change of postural direction will be so gradual as to render easy the conception of its reverse at the end of the journey.

Another and possibly easier way to reach this conception is by initial thought on the relativity of our notions of "up" and "down." Once we get firmly in our mind the idea that what we call "up" is merely the direction from our feet to our heads, and what we call "down" is merely the direction from our heads to our feet, we shall have no difficulty in realizing the conception that the man at the antipodes is standing with his head up as much as we are. An observer on another planet would find no reason to prefer our "upness" to his. Our heads and his are both pointed away from the earth's centre.

Huxley states somewhere that, while his reason assured him that our idea of the existence of external objects in time and space is a derivative concept and that our only immediate knowledge of such objects is as mental appearances, it was, nevertheless, impossible to conceive of them as mental only. It must be admitted that this is an extremely difficult conception, perhaps at one time impossible, but by no means so in the present stage of mental evolution. If one will concentrate profoundly on the necessary development of an infant or blank mind, he must see that the conception of the external existence of objects in time and space is a complex born of repeated experiences of such associated things as vision, perspective, touch, resistance, and motion and strain of the eye and other muscles. Abstracting these, and working backward, one is able finally to conceive of pictures of what in our later development we term external objects, existing, as to the infant, merely as mental states.

Perhaps the reader will pardon a digression from the main argument because of the deep interest of this question. It has been argued that the philosophy of the skeptic, who claims to believe in no existence other than that of his own mental states, may be annihilated by the fact that the very language of his contention must negative it, since there is hardly a word he can use which cannot be shown to assume the external existences he pretends to deny. Now the writer, in view of the analysis he has himself made in relation to our conviction of the reality of external existences, cannot be suspected of adopting the skeptical or solipsistic attitude. Nevertheless, he is satisfied that the argument just stated is not a valid one. It is true that the very language used by the skeptic connotes external existence as a reality, but it is equally true that language develops long before the mind reaches such a state of maturity as to be able to consider such questions at all; and, moreover, our physical structure is such that we have no memories of original infantile impressions.

The development may well be this. The infant in his earliest stage perceives mental pictures only. His experiences gradually lead him, in the way we have elsewhere analyzed, to posit them as external to his ego and to his own body, which latter also he posits as in a sense external to his ego, though partly within its control. Language comes later, and the infant gives names to the things observed and to their relations, all conceived as external to himself and in time and space. When his mind has developed to such an extent as to make of him a philosopher, he reasons backward through all these stages, so that he can quite comprehend how his original consciousness embraced no idea of external reality and even no idea of ego, but mental states only. His language is based on his later conceptions, the basic ones of all men, including philosophers, the conceptions necessary to existence in the external world, and that language could, therefore, contain little appropriate to the philosophic analysis later made. Philosophy, however, is not entirely helpless, as witness the invention of such a term as noumenon to explain the possible reality behind the phenomenal appearance.

To resume our discussion, we remark that, unlike mere difficult conceptions, there are certain *kinds* of notions which, it has been hitherto believed, can be in no way thought or symbolized. Man cannot conceive of a space with limits. If the conception of space itself be legitimate, a limited space becomes an impossibility. On the other hand, infinite space would require infinite time to posit. Therefore, it has been usual to say that, while we are compelled to think of space as infinite, we cannot by the very nature of finite mind picture or conceive it as such. These irreconcilable opposites have been generally deemed to show that the real nature of space is beyond human understanding. The same is true of time.

Even more convincing is the denial of conceivability to realities underlying phenomena. All the mind can directly know is its own states. From those states or phenomena it reacts in certain ultimate forms or conceptions, such as space, time and motion. It reacts in no form, however, to the noumena believed to underlie all these basic pictures and forms. They must forever lie beyond its reach, however great may be the number and power of new faculties we may suppose it, in the course of evolution, to attain. In no way could that veil be pierced other than by the conversion of finite mind into another and mystic sort, itself inconceivable, and with powers of a kind, not degree, removed from all present or past experience.

It is different with the phenomenal world. Here there may be and undoubtedly are vast regions, awaiting exploration and understanding, now unreachable by the human intellect, only because of the present lack of adequate sense equipment or such adequate environment as might supply the proper viewpoint. We are not now referring to those phenomena which, themselves imperceptible, are made, through experiment, manifest to our present senses, by their effects.

Here is man, with his few pitiful senses. His most vivid apprehension of things as he knows them comes from vision. Even more basically necessary to his existence are the senses of touch and muscular movement; while sound is of tremendous importance in the creation of a world of ideas unified through aural terminology, binding together multitudes of things by single names. Imagine him deprived of only one of these senses. If he had never had vision, imagine the dark lot, the stunted development that would have been his. Inconceivable to him, then, the luminous splendor of the visible world! Even with the explanations of a seeing being, describing to him in words the phenomena revealed by that sense, he would have been incapable of conceiving them. What countless other phenomena must there be, in the same way unknown to him through lack of the necessary senses with which to apprehend them?

Given man as he stands, even as certain of the motional rhythms of the restless universe express themselves to him in visual perception and the glories of color, certain others in sweet harmonies of sound, what a multitude of similar vibrations must exist, traveling through the vast reaches of space, which would reveal to him a universe of even greater and unimaginable splendor and variety, had he developed the additional senses adapted to their perception! The poet may dream of such symphonies, as yet unrevealed, trumpeted as though by angels from the deep recesses of infinity; but multiform as they may be, as strange, in their utter novelty, as light to a hitherto dark world, powerful and immense as the human faculties developed, of a nature to receive them, they would still be phenomenal only, mental appearances, analyzable into ultimate forms of physical motion, still concealing the profoundly mysterious and inconceivable reality which we are compelled, though indefinitely, to posit.

It is thus clear that we can conceive of the acquisition of new senses that may give us strangely new conceptions. To a man born blind and suddenly given the sense of sight, the revelation would, of course, be of a universe hitherto unsuspected and unknown. His feeling could scarcely differ, except in degree, from that of a man who, after dying a despairing death, should experience the unutterable joy of finding himself resurrected in or restored to a non-spatial existence, entirely inconceivable to mortal mind. And just as innumerable conceptions of the universe, now impossible, conceivably await the development of senses as new to us, as that of vision to our man born blind, so, logically, there may be conceptions as yet unborn even *in posse,* not because of the lack of the proper kind of sense apparatus, but because of an environment inadequate for the proper experiences, for the gathering of the necessary materials.

Euclid's geometry is the product of axioms, the contrary of which is not conceivable by us. Those axioms are born of our experience. Who can say, however, whether they would express such compelling truth, or all the truth, to beings living in a world of different experience? We react to the molar motion of ordinary bodies at ordinary velocities, in terms of time and space, to the molecular motion of relatively minute bodies at relatively enormous velocities, in terms of heat and sight and smell and taste. Who may say what other classes of motions there may be, beyond our present experience, to which we should react in forms now impossible of conception, but still conceivably conceivable?

But our possible new conceptions of things inconceivable now, we note, are constantly based on such ultimates, like motion, as we now know. If they are ultimates, we think all will agree, the underlying noumena are forever intellectually inaccessible. But are they ultimates? Does a deeper penetration and a greater generalization lie before us? That is the real question.

Einstein, admitting the present inconceivability of the products of his theory, believes that they may, nevertheless, be true. And so they may be. But if they are noumenal, their truth must be, like other metaphysical abstractions, beyond scientific demonstration. Science is based on the assumption of phenomenal realities, internal and external (which include such mental forms as space and time), that can be directly or symbolically pictured. Beyond that region lies that of speculation only. We cannot predicate qualities or magnitudes of an inconceivable.

Einstein claims that his admittedly inconceivable conclusions are or will be confirmed by experiment. His view seems to be that a thing may be mathematically abstracted from nature, which the mind cannot now picture; and that what is mathematically abstracted and empirically confirmed, is a scientific truth and not a metaphysical speculation. But if such truths are confirmed by experiment, they must deal with as yet unknown physical realities and hence are not really inconceivable, but merely difficult of conception, awaiting full mental realization when the new material is assimilated. Let us, therefore, closely re-examine our concepts of conceivability, in order to test whether Einstein's cosmology is or may be conceivable in any proper sense of the term.

So that the reader's concentration on this problem may be complete, let him imagine himself alone in space, in full possession of his present experience, but with nothing before him but a visible metal cube painted green, within his reach, his eyes, compulsorily, directed forward only. The cube and his memories and mental processes constitute his universe. He knows the complex sense impressions of the cube, which are internal, that is, mental only; he knows its greenness, those perspectives and shadows and outline, which, from past association with the sense of touch, present to him his visual impressions of solidity, sharp edges, shape and the like; impressions, too, which he can confirm by passing his hands over the object, sensing also by that act, the resistance to pressure, which will come on every contact whether he wills the contrary or not; from all of which and the sense of his own reaching motions is induced in him the conviction of the cube's substantial and persistent external reality in space and time.

Our observer's past experience assures him that if he could turn his head, he would discover an external source of the light which renders his cube visible. He knows that the greenness is mental only, and his scientific induction is that this mental state is due to cerebral motion, caused by the reflection of trillions of vibrations, in each second, of the light medium, which, traveling from their external source, meet the resistance of the cube surfaces and thence are diverted to his eye. He knows that whether his induction of ethereal vibrations be true or not is immaterial; that his visual impression of the cube is due to something traveling from the source of light to the illuminated object and thence reflected to him. All of the motions, except that particular variety which causes mental greenness, are absorbed as heat into the object's molecular structure; the particular variety, green, which cannot travel into the particular pigment of our painted cube, is beaten back by its resistance. Hence the object appears green.

Now, in this connection, all the argument in the world, and the most powerful conviction of the foregoing, will not enable the observer to *see* the cube as anything but green, so long as his eyes behold it. With sufficient reflection, however, you would not maintain that he could not conceive the truth. As a matter of magnitude, he would, of course, be unable to picture or conceive trillions, or even thousands of undulations in a second of time, but he could form some kind of symbolic picture thereof, sufficient, inductively, to enable him to demonstrate the physical laws of light. This conception is, however, that of a mental sight impression caused *by motion.*

In the same way, he can now press with his hands upon the cube, and, obtaining only certain mental impressions of resistance or muscular strain or heat or coldness, he knows that, while only these constitute certain knowledge, he can induce, from other motional experiences, that these impressions again are caused by material forces, by matter in motion. But note carefully that, while what he knows or may infer of sense impressions or ideas caused by matter in motion are true conceptions, it is not so with any concept he may attempt to form of the ultimate, *the motion itself*. This manifests itself only in sense impressions and derivative conceptual forms, but if we agree that motion is a physical ultimate, motion in itself, the mode of being which thus manifests itself, by the law of his nature, can never be present in his consciousness, can never be conceived by him. He has mental impressions of color, form, hardness, softness, resistance, and the like, all logically traceable to the same class of antecedent phenomena or causes which he describes as matter in motion in time and space, but motion is never known, never posited; only its effects. Motion, in itself, is as forever inscrutable as its derivatives time and space, always assuming them to be non-penetrable physical ultimates.

Our observer, after realizing that he can conceive that the color is not, as it appears, a quality of the object, but a mental effect caused by the motion of extrinsic matter reflected from the object, must come to the conclusion, on a similar investigation, and by parallel methods above sufficiently indicated, that the visible and tactual shape of the cube, its edges, sharp to touch and, by association of ideas, to vision, its hardness, the smoothness of its surfaces, are mental impressions only, from which he can infer and vaguely conceive matter in motion as the producing cause. But, no more than he can conceive the motion in itself, can he conceive the matter which is in motion, provided again that we agree that matter is a physical ultimate; and for the same reason, that is, because all that is mentally present are states of consciousness. Of course, in this discussion we leave out of consideration all such mystic mental processes, non-intellectual in their nature, which, some few believe, are now occasionally present, or may ultimately develop in the human soul, by means of which knowledge of the superphysical may be supernaturally realized.

Let us suppose our inquiring observer to pursue his studies further by breaking his cube to pieces with appropriate tools and continuing the work of destruction until the metal material is puverized into grains so small that each is hardly perceptible to vision. Let him now try the same process with a single grain. Mentally, even if he can find no tools of sufficient delicacy, he can divide the grain into two equal parts, each of such parts into quarters, each quarter into eighths, each eighth into sixteenths, and so on. If the material be a composition, let us say of copper and zinc, the process may theoretically continue until the molecule is reached, and the next division will then, of course, mean that the original material disappears as such, that is to say, is divided into its component atoms. Now let him proceed with one atom of copper. By the nature of thought he must deem it also divisible, and inductively he is satisfied that this further division will reach the electron. Can he then treat the electron in the same way, and if so, will he finally reach a simple corpuscle, perhaps the world substance or energy of which all matter is some sort of a complex, and beyond which further division is impossible?

Is matter infinitely divisible or not? To conceive that it is, is impossible, for it would require infinite time to complete the picture; but to conceive that it is not, has been said to be equally impossible, since by no process of thought can we picture a theoretically indivisible, finite magnitude. And yet, to use a strange expression, the last inconceivability is less such than the first. While we cannot possibly posit an indivisible particle, because so foreign to our race experience, we can vaguely recognize, in a fashion incomprehensible to our thought processes built on that experience, that matter may in its final analysis be simple and indivisible.

Let us now restore the cube intact to our observer and follow him in a further analysis of his universe. He posits his cube in space, his motions over its surfaces in space and time, its potentiality of energy, on being pushed by him, in terms of motion, space and time. The experience that he has of these phenomena reaching back for millions of years makes for him compelling concepts. All his ancestors, men and beasts, have been accustomed to the picture of a universe consisting of mind and of finite masses in infinite time and space. He proceeds to push his cube and climb aboard. He and his carrier will move, and there is no place that he can picture where his journey must of necessity end. He cannot conceive an end to the potentiality of motion. He may die, but others or imagined renewals of himself, may take his place and travel onward forever in time and space. He cannot possibly posit a wall behind which lies no further space or time, no further possibility of motion. On the other hand, it is equally impossible for him to conceive of unlimited time and space. The latter, however, is again, and strangely, less inconceivable, because while the race experience has never had and never can have experience of limitlessness, it has had and continually has experiences of boundaries beyond which it has never failed to find further expanses.

Our observer, let us say, is of the opinion that Einstein's views are unsound. He does not believe it possible, when he has pushed his cube, that it will move other than in the ordinarily conceived way, in space as ordinarily conceived. But he is a conscientious fellow, and admits to himself that perhaps he has this firm conviction because neither he nor any one else has ever experienced anything but ordinary velocities. He remembers that Einstein's theory requires extraordinary speed to make its effects appreciable. He conscientiously reflects that it is not strange that he cannot form even the vaguest picture of a curved or twisted space or of matter doing that in it which we call "moving," having nary an ancestor who ever had such an experience or thought. He invents a process by which he gives his cube a velocity of 161,000 miles a second. Off he goes on the great experiment. He finds no change; that is, as his experience would be reported to us by the relativists. Everything will be the same as before, unless he should happen to pass another system inhabited by a man like himself. To him this man would exhibit strange differences unknown to the man himself, and perhaps physically explicable. But suppose the view of the relativists is incorrect. Suppose he were able to perceive a fourth dimension. Then it would be conceivable to him that if his ancestors had always lived under such conditions, he might find it as impossible to conceive a straight and invariable space and time as he now finds it to conceive them mixed together.

These are difficult waters in which scientific men have hitherto declined to swim. We shall presently explore them a little further. How difficult they are will appear from one more reflection of our observer on the green cube, which may determine him to cling to such conceptions as he has of his own universe, already sufficiently mysterious, beyond all human understanding. He has been supposing himself to be moving through a space devoid of all other bodies. He has not encountered the other system. But, under such conditions, is he really "moving"? There is in him and his cube or somewhere, since the push, some active principle. But is it motion as theretofore conceived by him? Certainly not. He will be as entirely unconscious of such a thing, in the absence of other bodies with which he may compare his position, as all men are of the motion of the much-moving earth. Motion of one body alone in a boundless and empty space is entirely meaningless, since the conception is concerned alone with change of position relative to boundaries or other bodies. Our observer remembers, however, previous motion in relation to such other bodies, and their motions in relation to himself at rest. Even though he knows of no experience of any body which, on investigation, turns out to be really at rest, he believes himself compelled by his mental processes to think of such absolute rest. Perhaps this is only because the race has ever experienced motions which appear to be such in relation to a stationary observer.

An ancient hurler of the javelin thought of himself as of fixed position. The earth's multitude of motions carrying him with it, was certainly not present in his consciousness, and practically is not to his modern counterpart, even though the latter is better acquainted with the facts. But whether or not this apparent stability be the sole genesis of the notion that there must be actually some stationary body, or, at least, an infinite number of imaginary stationary points in space, in relation to which bodies in motion are actually moving, this notion in our observer, as in all of us, will not down. And yet it also is entirely incomprehensible. We cannot conceive of an infinite number of points, nor even of a single stationary point.

A point is a mathematical abstraction, and for purposes of comparison with a moving body, we must posit a physical point, that is, a mass or body. But the ascertainment of the state of motion between two bodies can give us no knowledge that is not relative, according to the situation of the observer. Thus, while compelled to think of actual motion in empty space, we cannot understand it. It is the case, over again, of the alternate impossibilities, in thought, of limited space and time, and unlimited, of infinite divisibility of matter and of limited divisibility.

The foregoing analysis establishes, we believe, the following notions as inherently inconceivable: infinite time, infinite space, limited time, limited space, infinite divisibility, and noumena. It will hardly be necessary to discuss further any of these except the last. So far as concerns infinite time and space they will hardly enter into our present problem for reasons that will shortly appear.

Noumena are the unknowables underlying the ultimates in the phenomenal world. While it is true that what may be an ultimate to-day may cease to be such to-morrow, it is also true, in spite of the increasing generalizations of science, and in its latest as well as in its earliest stages, that the most searching analysis has never been able to dispense with a few basic conceptions which have defied further penetration. These are matter, energy, space, time and motion.

The race experience is only of phenomena analyzable into matter and energy and the manifestation thereof by motion in time and space. The concepts of time and space are derived from that of motion. Behind all of them there is an indefinite consciousness of a noumenal, and hence unknowable, force which is revealed in different modes through these ultimates. But science has been progressing more and more, and recently with tremendous strides, toward the unification of phenomena as manifestations of one basic, physical reality. Though the inductive proofs of this are now accumulating, it is the crowning achievement of philosophy that, almost from its beginning, it has anticipated this result.

The ancients formulated the idea of elemental atoms, which science in our own era has demonstrated. But long before the recent electronic discoveries, philosophy had prophesied and eagerly looked forward to the further analysis of the atom and the demonstration of the basic sameness of matter in all its elemental differences. The recent investigations seem to be demonstrating that one element differs from another only in some such simple way as the number of electrons revolving about the central proton, electrons and protons alike being reducible to forms of electric energy. Philosophy had long since posited matter as a manifestation, mentally, of energy, though in any view both are ultimates equally incomprehensible.

Philosophers and scientists alike have been prognosticating the ultimate demonstration of the evolution of atoms and their constituents from the mother ether. Knowledge has not as yet penetrated thus far, but the vague thought is that in some way the protons and electrons are the offspring of the ether, developing from it in the form of mysterious knots or vortices or the like. The theory that it is the seat of electric energy is, by this time, more than merely plausible. The idea has even been advanced that in some subtle, and as yet vaguely comprehended way, gravitational force may be traced to some mode or activity in its all-pervading substance. The reality of its presence seems manifested in such phenomena as light, heat, electricity and magnetism. It apparently has no weight, but if it be concerned in the production of gravitation, it is no violation of classic ideas to suppose it a kind of matter or substance, immune from that which it produces. Weight is not necessarily a quality of all matter, but a force or influence to which it is subjected in given conditions. But, however the ether be conceived in relation to the phenomena which gave it mental birth, it is in the usual terms of substance, energy, motion. It is a more than respectable hypothesis to suppose it in some way the mother of all physical energies. Thus, however far science has advanced, alike in our own time and in the times when the acquired scientific material was relatively meagre, each new generalization and each new phenomenon contained within it, however inconceivable before, have always been finally conceived in terms of the same ultimates. Moreover, as far as we are reasonably able to forecast the future progress of science, we cannot conceive of new discoveries except in those terms. Until Einstein, at least, no thinker would have claimed the possibility of phenomena except as involving matter or energy, static or moving, in time and space.

We derive a sensation like color in a way incomprehensible to us, from physical vibration. The rapidity of the vibrations is inconceivable only in the sense that symbolism must be brought to our assistance. It is the same with the new electronic discoveries. The idea of trillions of electrons within a drop of water, each separated from its proton by the same relative distance as a planet from its sun, and revolving about it at relatively enormous velocity, is, of course, picturable only symbolically; but all such concepts are still those of energy, matter and motion, and, however incomplete, sufficient, with the necessary mathematical relations, to result in mental comprehension and prediction of effects. It is true that in the electronic world certain divergences from orthodox expectation seem to have been manifested, but our knowledge has as yet hardly done more than touch this field, and even these phenomena are explicable on classic standards.

The atom, which may not be directly perceived, became a definite scientific conception, following the discovery of the laws of definite and multiple proportions, which it alone could explain. It immediately became picturable as matter in motion in space and time. While the writer is not called upon to strain his meagre physics by undertaking to describe the experimentations that have revealed the electron and other recently discovered secrets of nature, it will suffice to say that this kind of investigation is of the same general character as that which, in the beautiful arrangement of iron filings in a magnetic field, discloses some of the ethereal secrets. Thus, indirectly by effects, the cause may be pictured. But all these new concepts are of motion in time and space; and however symbolical they must of necessity be when one realizes the unimaginable complexity and immensity of their subject matter in its relatively minute environment, they and their relationships have always yielded, so far as knowledge yet extends, to reasoning and calculation of an orthodox character. The ultimates are always the same.

It might be asserted that behind these physical ultimates, for example, behind the atomic motion that causes such a psychologic ultimate as color, lies a Being that thinks these psychologic ultimates, that thinks color, and transmits it through the instrumentality of physical motion to our material brains, where it is transmuted to its original state by a mentality that is part of the same Being immanent in all nature. Such a Being, however, must be noumenal and this notion is, therefore, a pure metaphysical speculation of a pantheistic nature and beyond the domain of science.

On the other hand it might be asserted that, if we were differently constituted or were set in a different world environment, the same motions to which we react in vision and color might conceivably cause reaction of a different and now inconceivable kind. This, as we have seen, might well be true. Indeed we have direct evidence of its truth. Besides the rays that cause vision, there are those which cause heat, and others to which there is the chemical reaction of photography. Only in recent years others, never known before, have been indirectly, by experimentation, brought within our experience, such as the ultra-violet and the X-rays. It is easily conceivable that, under proper conditions, our vision might have used the latter and been able to penetrate objects ordinarily regarded as opaque.

In the same way, it might be asserted that, if we lived in a different world with different brains, that is, under conditions now not even imaginable, our minds, which now react to certain phenomena in concepts of motion in time and space, might react to them in quite different and now inconceivable forms.

The natural attitude is one of rebellion against this claim. It is an attack upon the fundamental concepts of a race experience of æons of ages. It seems to question the basic conditions which lie at the root of all our being and all our thought, and no less so for the philosopher than the common man. But, as was said, I believe, by Lodge, our sense experience was evolved by the age-old struggle for existence, and not by the relatively recent conditions of philosophic thought. The subject is entirely too difficult for dogmatism. We must try to answer, as best we can, whether there is anything in the Einstein formulation which indicates a further analysis of any of our ultimates into non-ultimates, so that either of them becomes a manifestation of a new ultimate, which is, therefore, not noumenal but phenomenal.

Let us examine first the basic product of the special theory. We have seen that, according to the relativist idea, a body in motion in no way exhibits the Einstein changes to those who travel with it, while each such body to observers on another, will, if the velocity be sufficient, manifest decided changes in lengths, mass and time, hardly explicable on any theory involving classic standards. If the relative velocity be increased, when it has reached, or rather, is about reaching the limiting velocity of light, to the observer on each body, time, on the other, will be ceasing, the dimension of the observed body in the direction of travel will be growing infinitely thin and its mass infinitely great. The earth might be traveling at 185,000 miles a second relative to some now unknown body. According to the relativists, we should not know it, but to observers on that body we should be approaching physical annihilation. Our thinness in the direction of travel and our mass would be approaching the infinite. And in spite of our happy ignorance, these changes would be, in some sense, actual.

This, of course, is in every way inconceivable. It involves not only infinities, but contradiction in thought. And if we are not to rely on our mental processes, we must cease to philosophize. We become helpless. But suppose the explanation is, as we have before indicated, that the observations from relatively moving bodies, while recording a real condition according to the point of view, do not contain the actual activity. Suppose that the observer is perceiving, mentally, motion in time and space, but that what is actually taking place is what until now would have been deemed an inconceivable activity, in a union of space and time, to some extent at least, affected by his own principle of motion. He is then deriving, with deficient instruments, his own interpretation of phenomena really occurring in a four-dimensional world. In our ignorance of that real world, we can no longer speak of space or time or motion or infinities. The trouble may be that our point of view, our environment, our sense apparatus, are all pitifully defective. In that view, can it be stated positively that it will always be impossible to conceive this real world, until now manifesting itself to us in motion in space and time, any more than it has always proved itself impossible to conceive of color as the mental product of physical motion? We are not alluding here to the incomprehensibility of the transmutation of cerebral action into mental forms. To attempt to explain that would be indubitably to attempt to penetrate into the noumenal world. We are alluding to the fact that at a certain stage of human knowledge it would have been impossible, though not inherently so, to conceive of motion as an element in vision.

The best answer, then, that the writer is able to make to the great question we have been considering, is that it is not impossible, logically, that motion, space and time may not be phenomenal ultimates. It is not impossible that the physical reality of a space-time world may underlie them.

We give this answer of logical possibility, however, with misgiving and reservations. It should not be forgotten that the reduction of the vision phenomenon of the illustration just used, as of all others, is to phenomena of motion in space and time. We are utterly without experience of any deeper ultimate. The space-time world idea is a denial of motion and a denial of space and time. It is, therefore, a denial of everything that is basic in human thought. We are utterly incapable of thinking its pictures. While, therefore, we state our belief in the possibility of the ultimate realization of this conception, we assert with much more confidence that the burden of proof upon him who asserts it is as tremendous as science would rightly impose upon him who asserts a miracle. We assert, moreover, that if the analysis we have made in Chapter VIII of the time concept be valid, the concept of a physical space-time ultimate becomes all but impossible.

If the space-time world is noumenal, it is forever inaccessible to men. If it is phenomenal, it will never be really comprehensible to them, until their Einsteinian studies have developed another sense or a new form of mental reaction. Moreover, even conceding its possibility, it may nevertheless be far from an actuality. And finally, if it be actual, we think the pragmatic position must be conceded, that truth must forever elude us, since we can assert of no phenomenon that it is physically ultimate. Our point of view must be forever false.

CHAPTER X

THE POSSIBILITY OF A GENERAL THEORY OF RELATIVITY

NEWTON'S cardinal law of motion states that a body at rest will continue at rest and a body in motion will continue in motion in a straight line until either state of rest or motion is changed by force. This law formulates that quality of mass which we know as inertia, which, crudely expressed, implies the resistance which a body offers to a change in its existing state, whether of rest or motion. The amount of this inertia or resistance in a body is in proportion to its mass, which, in turn, may be expressed to be the quantity of its matter, an expression, however, which always has left much to be explained, and now, more than ever, in view of the recent researches into the ultimate nature of matter.

When the force exerted upon a body to change its state of motion is gravity, the inertia becomes, in a sense, heaviness or weight.

This follows from the law that the gravitational mass or heaviness of a body is the equal of its inertial mass, that is to say, the equal of its resistance to a change in its state of motion from some force other than gravitation. That law has been recognized before Professor Einstein. He offers an interpretation of it which we shall presently note.

Newton's law of motion is not susceptible of direct proof in experience. We have no authenticated case of a body absolutely at rest, and certainly none of a body moving on forever in a straight line. The law is confirmed, however, by proof that the motions we do know are its precise resultants after allowance for the forces impressed. If two or more forces together operate upon a mass to produce motion, the product, according to Newton's law, is motion in a straight line, of a velocity and direction which are the resultant of the co-operation of the producing forces. Our universe is a complex one; its forces, whatever they may prove to be in a last analysis, are, for us, many and not one. Newton's law means that, after their operation in a particular case, the product or resultant, in direction and velocity, is final, so that the motion thenceforward must be rectilinear and uniform unless and until again interfered with. The addition of force to force adds acceleration to motion and destroys its uniformity. If, as the resultant of one or more forces, a body is propelled in a certain direction, at a given velocity, its condition of motion is permanent and uniform until other forces intervene. If the intervention be applied at an angle to the existing motion, a new resultant will arise.

If the same force be continuously applied in the same direction, if, in other words, the force applied be not merely a single but a continuing push, that is, a continuing series of pushes, each push is added to the other, so that the motion of the body becomes accelerated with a constantly increasing velocity. If the newly applied force or resultant of forces is applied in an opposite direction to the existing motion, the result will be a constantly increasing retardation. Any change in the direction or quantity of motion is termed an acceleration, a positive acceleration when the velocity is increased, a negative acceleration when the velocity is reduced. It needs little physical or mathematical learning to realize that when a constant acceleration is applied to an existing state of rectilinear motion at an angle, the result will be motion not in a straight, but in a curved line. The acceleration expresses itself in the constant change of direction of the curve. The most familiar case of this, as in fact the most controlling fact in the mechanism of our universe, is the constant operation of the force of gravitation, with the resultant travel of planets and their satellites in elliptical orbits.

In order to make clear the more important part of the foregoing to those who may have forgotten some of their physics, we shall interrupt the course of the exposition with an illustration. Let us suppose that the earth, its atmosphere removed, were the only body in space, or, at least, so far removed from other bodies as to render their influence negligible. Then let us suppose that a ball were fired from a cannon in a horizontal direction from the top of our highest mountain peak. Everybody knows that this ball would ultimately reach the earth after travel in a curved line. This is the ordinary resultant of the two forces in play. The first is that at the cannon's mouth supplied by the discharge. The discharge accomplished, the cannon's work is done and the ball's motion would be uniform in velocity and unalterable in its horizontal direction forever, were it not for the second force, the constant, pulling, gravitational attraction of the earth. A pull from the earth, so to speak, is added to each preceding one and gradually brings the ball to the ground in a curve. If the ball had been at the cannon's mouth in a state of rest when subjected to this pull, the result would have been a constantly increasing or accelerated motion perpendicularly downward.

In a vacuum, the earth pulls all bodies near its surface, whether heavy or light, with a constant acceleration in each second of approximately 32 feet per second. That is to say, the distance traveled by any such body when started from a state of rest will be, in the first second, the average of the rate of acceleration or 16 feet, and its velocity at the end of the second will be 32 feet per second; in the second second the distance traveled will be the velocity of 32 feet at its beginning plus 16 feet or a total of 48 feet, while its velocity at the end will be 64 feet per second; in the third second the distance traveled will be 80 feet, and so on. Of course, theoretically, the object pulled attracts the earth in the same way that the earth attracts the object, but in the consideration of masses relatively so unequal, the pulling force of the smaller may, for practical purposes, be disregarded.

Let us now suppose that men have invented a machine for discharge of projectiles at velocities much greater than have been heretofore possible and that these projectiles are now fired in the same way as our cannon ball. If the velocities are sufficient, that is, if they range from about 5 to 6½ miles per second, the projectiles will, in each case, clear the earth, and, as a resultant of the two forces in play, continue their respective journeys in a curved line about the earth, returning to the starting place; and, if the high position from which they were fired conveniently disappear, continue thus indefinitely, without earthly contact. If the firing force were such (a little less than 5 miles per second) as to reach, with the gravitational force, the precise balance necessary for the purpose, the ball would continue in a perpetual circle around the earth. At greater initial velocities, the course of the ball will be, not a circle, but an ellipse,—that beautiful form where, due to the varying velocities of the ball as it nears or recedes from the focus, the area included between the orbit and two lines drawn from the focus to the ball at any two points of its journey, will at all times be equal to a similar area included between two other points of the ball's travel in the same length of time. As the chances, in nature, of the precise balance necessary to secure a circular orbit are remote, the orbits of the planets are elliptical.

If now our invention were improved, velocities might be reached so great as to enable the projectile to escape forever from the gravitational pull, so that it would proceed in parabolic or hyperbolic curves, rounding the earth, to be sure, but out into space, never to return.

The foregoing illustration gives some notion of the manner in which gravitation acts. Mathematically, the law, as formulated by Newton, is that bodies attract each other in direct proportion to their respective masses and in inverse proportion to the squares of the respective distances between them. To this, in order to possess sufficient data with which to follow the Einstein argument, we need only add that cardinal characteristic of gravitation, to wit, that the attraction acts on all bodies alike, irrespective of their kind or quality, and irrespective of the intervention of other bodies. As is well known, magnetic forces act between particular kinds of bodies only, and the magnetic force may be interfered with by particular interventions. Gravitationally, however, every body attracts every other and does so notwithstanding any interventions whatever. The sun attracts the earth, and the earth the sun, even when the moon intervenes.

The cannon ball illustration, extended, gives a clear picture of the manner in which the solar system, in its larger aspects, is constructed. The planets revolve about the sun and the various satellites about each planet in elliptical orbits; and whether the initial centrifugal impetus of the revolving body was imparted, as is assumed by the nebular hypothesis, by the flinging off of rings or other forms of gaseous matter by the axial rotation of the central body, or in some other way, this initial impetus added to the gravitational pull is sufficient to account for the structure and motions of our solar system. That system, then, comprises bodies, each of which contains within it the power of gravitational pull. While it is true that each member of the system attracts every other, practically we may picture each, relatively large as compared with the relatively small bodies that are attracted on or near its surface, as having about it a gravitational field, the intensity of which increasingly diminishes, as above stated, with the distance from the attractive centre. The earth has a gravitational field by which bodies relatively near its surface are impelled towards it in each second at an acceleration of 32 feet per second.

Such being the general nature of our universe it is clear that the special theory, limited as it is to uniform motions in straight lines, must be extended to cover the phenomena of acceleration, if it is to have universal validity. According to classic mechanics, relative acceleration between two systems involves nothing other than more complicated transformations. The special theory, with its Lorentz transformations, involves a more difficult problem. We may regard that theory, as we saw in Chapter VII that Einstein does, as the product of the relativity of motion and the primary law of light, or we may regard it as the product alone of the hypothesis of the secondary law of light evolved from the Michelson experiments. In either aspect the actuality of motion is denied by Einstein.

As this principle of relativity, or, as Einstein would put it, of the non-actuality of motion, is necessary to the special theory, it becomes impossible to extend the theory unless we can extend that principle to accelerated as well as to uniform motion. The first part of Einstein's remaining problem, therefore, consisted in the determination of the possibility of that extension; the second and final part, that possibility established, in the mathematical formulation of the extended theory.

In beginning the discussion of the possibility of the extension, he reverts to our old friends, the train and embankment, and asserts that, at first sight, to an observer on the train, it would seem impossible to apply the special theory to such a state of accelerated motion as should be caused, for example, by a sudden application of the brakes. In such a case, the observer would be flung forward, due to the negative acceleration of the train. Could he continue to believe that the embankment had been moving, and not the train? An affirmative answer seems to the writer self-evident. We are supposing the train to be moving with a perfectly uniform motion, of which the observer on the train is as unaware as all of us are of the earth's motion on its axis. Perfect uniformity of motion being impossible in earthly experimentation, perhaps the picture will be better if we suppose our observer's vehicle not to be moving on an embankment, but flying through empty space. In that case, his jerk forward, due to a sudden negative acceleration of his former motion, might just as intelligibly be attributed by him to some external force as to an internal one. He could validly interpret it as being due to collision with some other body, or to some force in front attracting him or repelling the train.

Professor Einstein, however, evidently does not regard this solution as self-evident since he discusses it at great length. In that discussion, as well as of the questions that follow in connection with the development of the general theory, his method ceases to follow the forms of logic, and becomes illustrative. The first illustration, though it carries many instructive implications, is originally for the purpose merely of solving this question of the relativity of accelerated motion, or, concretely stated, the problem of the train and the brake. This is the well known and beautiful illustration of an experimenting scientist in a spacious box or chamber, alone in a space empty of all bodies, other than one non-interfering body on which there is another observer.

Fully to comprehend this illustration, the reader must remember that the force of gravitation, so common in earthly phenomena as to render it difficult to dismiss it from the mind, is entirely absent, and that the imaginary experiments are all performed in a vacuum.

Owing to the absence of gravity, it cannot be assumed, of course, that either side of the box is floor or ceiling. We will designate, however, as the floor that side upon which the scientist happens to be standing. To maintain his position there, he should fasten himself to the floor, because, otherwise, the slightest impact upon it, even that of a muscular jerk on his own part, would send him traveling toward the ceiling. He removes from his pocket a small object and releases his grasp upon it. It does not "fall." It stands in space precisely where he has released it.

Let us suppose now that, unknown to our scientist, a rope is fastened to the centre of the outside of the ceiling, which is the side of the box opposite the floor, and that some force maintains a constant pull on this rope. The chest, of course, moves with an accelerated motion in the direction of the pull. A single pull would result in motion in a straight line at uniform velocity, but its constant maintenance results in motion in a straight line at a constantly accelerated rate. If the rate of acceleration be precisely that of gravity at the earth's surface, the box travels at the same accelerated velocity with which bodies gravitate toward the earth. The difference is that on the earth, the motion is finally arrested by contact with the earth, while, in the case supposed, the motion increases as long as the pull continues.

We take it that no one will doubt that, under the circumstances depicted, the scientist is absolutely in ignorance of any motion. There are windows to his box, but, in the absence of outside objects for comparison, there is no way of detecting his motion, and, assuming their existence, he could not be certain whether his box were in motion, or they. After the box's motion has begun, our scientist releases from his grasp various objects of different quality and constitution, and observes, with the aid of apparatus, that each of these objects travels to the floor at the same rate of acceleration as in previous experiments on earth. This is true, even after the box has attained a high velocity, because each object, at the instant of its release, already has the velocity of the box and man at that instant, and the floor therefore appears to travel to it at the velocity of the original acceleration. While the motion, from the point of view of the external reference body, is that of the floor of the box towards and against the released objects, the appearance to the scientist is precisely the same as though the objects were falling to the floor in a gravitational field, and his certainty of this is the greater when he notes that objects of the most diverse character fall with the same velocity.

He comes, therefore, to the conclusion that he is in a gravitational field, and wonders why the box itself does not fall. He climbs through a window for the purpose of investigating the exterior of the box, observes the tautly stretched rope attached to the centre of the ceiling, and, of course, decides that his box does not fall because it hangs in a gravitional field, suspended from some distant and invisible structure.

To the writer, this illustration as thus far carried does little more than accentuate our dense ignorance of gravitation, and it serves, therefore, like so much else which Einstein writes, as a caution against preconceived notions. While gravitation appears to act as a pulling force, it may be that the reality is entirely different. It should not be forgotten, however, that the illustration is purely fanciful. Einstein himself cautions his readers not to suppose that a gravitational field could in every case be assimilated to the phenomena caused by the accelerated motion of the box. Obviously the earth could not so be assimilated for many reasons. We need only point out that, if its gravitation were due to its motion against the apparently attracted objects, there not only could not be the manifestations of gravitational attraction on the opposite side of the earth, but, there, the objects released would appear to fly from and not toward the earth.

As we have remarked, the use Einstein makes of this illustration is to derive from it the relativity or non-actuality of accelerated motion. After proceeding thus far, he says:

> "We have thus good grounds for extending the principle of relativity to include bodies of reference which are accelerated with respect to each other, and as a result we have gained a powerful argument for a generalised postulate of relativity."

But the illustration is, of course, no proof of this extension of the principle of relativity, because Einstein, in the course of the illustration, has assumed, as we have done, that the relativity of accelerated motion is self-evident. He has assumed that, in the same way as a uniformly moving observer may interpret himself at rest and stationary bodies in motion, the man in the accelerated chest may deem himself at rest and the released objects in motion.

Before proceeding further with Einstein's development of this idea, and because to it he joins some others derived from further experiments of the scientist in the chest, we will first describe those further experiments. The scientist attaches a rope to the centre of the inner side of the ceiling and fastens an object to its free end. Such a performance on his part, before the box had begun to move, would have resulted merely in the rope hanging in such loose way, and the object maintaining such position as those in which he had happened to place them. But, the chamber moving, the rope is stretched taut and hangs "downward." We, of course, use the word "downward" to indicate the man's notion of up and down caused by the assumed presence of the gravitational field. The object attached to the rope appears to him to hang from it in the same way as we are accustomed to see bodies hang when suspended in gravitational fields; that is, to him the attached body appears to be experiencing a downward force in the field, neutralized by the tension of the rope.

Our other observer in space noting the motion of the box, now sees the experiment being conducted in the interior with the rope and its attached object. To him there is no suggestion of a gravitational field, but the box is seen to be moving at an accelerated velocity. If the constant pull of the external rope continues indefinitely, of course, the velocity will for him reach unheard of magnitudes. To him, the action of the object attached to the inner rope does not appear that of a mass gravitationally attracted to the floor, but his interpretation is that the rope, having to take part in the accelerated motion of the box, transmits this motion to the body attached to it, the tension of the rope being just sufficient to effect the acceleration of the attached body. To the man in the box the magnitude of the rope's tension is the gravitational mass of the suspended object, while to the observer on the outside this magnitude is its inertial mass.

Merely because in an illustration of this kind the same phenomenon is equally capable of these two interpretations, Einstein draws the conclusion or at least derives the hypothesis, not only that inertia and heaviness or weight are identical, but that they must of *necessity* be so. He says:

> "Guided by this example, we see that our extension of the principle of relativity implies the *necessity* of the law of the equality of inertial and gravitational mass. Thus we have obtained a physical interpretation of this law."

That interpretation is his famous Principle of Equivalence, the importance of which to the general theory, we shall later have occasion to note. Concisely put, it means that the same quality of a body manifests itself from one point of view as heaviness, and from another, as inertia. We shall have more to say on this subject in the next chapter. We have anticipated here, in order to follow the order of Einstein's exposition of the chest experiments.

His interpretation of those experiments as solving the question whether the man in the moving train, retarded by the application of the brakes, could still consider his train as being at rest, and the surrounding objects in motion, is as follows: It is true that he experiences a jerk forward when the brake is applied, but he is not compelled to refer this jerk to a "real" retardation. The illustration of the man in the box shows conclusively that the train observer might equally consider that he is at rest, but that there exists with reference to himself and the train a gravitational field directed forward and variable in time, that is to say, a gravitational field that operates only from time to time; at such a time, for example, as during the application of the brakes. When this field operates, he might validly consider that he is attracted forward at a non-uniform velocity and that the embankment is also similarly attracted forward, so that its original velocity in the backward direction is continuously reduced.

The principle then, being established, at least as a plausible theory, that all motion, even accelerated, is relative, and, according to Einstein, never actual, it follows that the special theory may be theoretically extended to the phenomena of the actual universe. The special theory states and means that, with the assistance of the Lorentz transformation equations, all systems at rest or apparently so, and all moving uniformly in straight lines or appearing so to do, are equivalent for the description of natural phenomena, that is, for the formulation of natural laws. The general theory is tentatively stated to be that all systems are thus equivalent, whatever may be their state of motion. The next question must therefore concern the possibility of applying the Lorentz or similar transformations to the phenomena of accelerated motion. We shall see in the next chapter that the slightest examination of that problem will necessitate a rewriting of the formulation of the general theory just given.

CHAPTER XI

THE GENERAL THEORY OF RELATIVITY

IF the idea of relativity as embodied in the special theory is to be successfully extended to accelerated systems, that is, to the world as it actually exists, our first inquiry must be as to the mathematical method of transformation to be used. We examine the Lorentz transformation equations and note that they contain a constant c, the velocity of light, and the various co-ordinates x, y, z and t, whose magnitudes on one system are to be reduced by the equations to those on another. If, under the influence of gravitational fields, the velocity of light is accelerated and thus ceases to be a constant, the method of the Lorentz transformations will naturally be no longer available. The question, therefore, is whether light is subject to gravitation, for, if it is, the constancy of its velocity, which was a fundamental assumption of the special theory, disappears, and the mathematics of the general theory cannot depend upon that quantity.

Moreover, it is sufficiently evident that, if the special theory be valid, we can no longer rely upon Euclidean geometry. It may be that for the simple conditions of a universe containing only uniform, rectilinear motions, the Lorentz equations will satisfy the requirements of transforming phenomena as viewed from one system into their appearance as viewed from another. But, as we have seen, the outcome and explanation of the special theory was found in a four-dimensional world; and it is manifest that in such a world and under the influence of the complications introduced by rotary and otherwise accelerated motions, we can no longer rely upon space and time measurements in the usual terms of linear and time units. If then we cannot give definite magnitudes to our x, y, z and t quantities, we can no longer use a simple mathematics like that of the Lorentz equations. These considerations sufficiently indicate the non-Euclidean nature of our problem. But, as the Professor considers it necessary to establish this point further, we shall follow him in his exposition. His method in relation to this matter, as well as in relation to the question of the influence of gravitation upon light, is, as throughout his presentation of the general theory, illustrative.

In relation to light, he returns to the scientist in the chest. Let us suppose that an object travels, at a considerable distance above that gentleman's head, across the spacious interior of the chest, and, to the outside observer, parallel to the floor. It will require only a moment's consideration to realize that the accelerated motion of the chest will cause the scientist to perceive this traveling object as falling to the floor in a curved path, precisely in the same way as the cannon ball of our previous illustration traveled toward the earth.

Einstein now assumes a ray of light traveling in the same way as the object just assumed, and calls attention to the fact that, to the scientist in the chest, this ray will similarly appear to fall to the floor in a curved line. He says that this is "a new result of fundamental importance," and he concludes from this that (the italics are his) *"in general, rays of light are propagated curvilinearly in gravitational fields."*

This sudden transmutation of the result of a fanciful illustration and of what may be merely an illusion into a law of nature is startling. But this is all that can be derived from the Professor's exposition. He resorts to no reasoning. He assists us in no way to comprehend the deduction of so important a law from what apparently is so flimsy a premise. On reflection, however, it is quite evident that he has derived this conclusion by adding to his illustration some very large assumptions. He assumes the non-actuality of motion, the existence of the space-time world and his principle of equivalence. Supposing the ray of light to be visible, there is no question that, to the man in the chest, it will appear to travel to the floor in a curved line, precisely in the same way as the cannon ball appears to travel to the earth in a curved line. For him the ray will appear thus to be falling in a gravitational field. It is equally clear that, to the observer on the outside system, the phenomenon will be an ordinary case of inertial motion. The fact that the one phenomenon appears in two forms by reason of the difference in viewpoint certainly does not amount to a demonstration that in a natural and relatively stationary, as distinguished from an artificial, gravitational field like the moving box, the moving light will fall in a curved line. Thus it is far from proving that, on earth, light has weight.

If, however, the principle of equivalence be valid as a general law, then one can deduce from it that, because what at bottom must be the same phenomenon appears to one observer as inertial and to another as gravitational, there must be a real phenomenon in the real world that lends itself to these different interpretations. Since the actuality of the motion of light, as of the motion of the box, is denied, since it is assumed to be as true to state that the light is moving toward the floor as that the floor is moving toward the light, the reality must be some condition of activity which causes to both observers the appearance of motion, to one the motion of the box and light, and to the other the motion of the light. But if there be such activity which is not actual motion in either, we are again reduced to the supposition that it must be in the common medium, in other words, in the space-time world.

We thus see that on the basis of the scientist-box illustrations of the last chapter, the Professor has extended his principle of equivalence into a general law. Of course, he has done this as a working hypothesis only, to be justified solely by empiric proofs. That law fits in well with the idea of the space-time world which we have endeavored to explain in Chapter VIII. It is in harmony with the idea of relativity. It means that "real" phenomena appear to different observers in different ways, according to their respective conditions. Motion is not actual. Gravitation is no force. There is in fact no actual motion-producing force of any kind. Force is relative. The appearances of one kind of force overcoming inertia and causing centrifugal motion, and of another kind causing gravitational motion are in reality merely indicia of what at bottom is one as yet inconceivable phenomenon.

We are now able to see, for the first time, what we must come to realize is of prime importance in the theory, that all these apparent forces have been wrongly thought of as such, and that *what we have to do with is not forces but structure.* The structure is, of course, that of the four-dimensional world. The appearances to us of the phenomena in that world are determined by the way in which we view, from our standpoint, its geometry.

But this geometry is subject to change. We have seen that the idea of the space-time continuum includes a notion of some inconceivable twist or curvature corresponding to the time dimension where there are bodies which endure, but we are now able to see that this continuum, already twisted, is subject to such further twist or curvature as is caused by that principle of energy or activity which expresses itself to us in motion, where bodies move. The amount of this secondary twist, if we may so term it, is dependent upon the point of view, upon the relative differences of velocity involved; for example, upon the intensity of the gravitational field and the relative velocity of the observer. The influence of the field upon light, expressing itself in the curvature of the ray, depends upon the intensity of the field and the velocity of the light. These are not actualities, however, but appearances only, and the real phenomena are activities in the four-dimensional world which make themselves apparent to us as phenomena of light travel and of gravitation. These activities cause "travel" along the space-time lines of the four-dimensional world, according to its *non-Euclidean geometric structure.* This inconceivable travel is guided by that structure, and appears to us as ordinary motion in space and time or as mere stability in space and endurance in time. To put the matter in another way, the real activities are those of matter and energy, and these *make* the space-time structure according to their needs.

The supposed law of the curvilinear path of light rays in gravitational fields is stated by Einstein to be of great importance in two respects. In the first place, it can be compared with the reality; and in this connection he made his famous prediction of the discovery of curvature in light rays passing through the gravitational field of the sun, from stars near its edge, that might be observed in times of solar eclipse. He predicted the direction and quantity of the deflection of a star from its usual position, under such circumstances. This prediction will be mentioned again when we take up, in the next chapter, the matter of proofs of the relativity theory.

We shall then discuss this matter of gravitation and light a little more fully. It should be stated here, however, that physicists have long contemplated the possibility of the susceptibility of light to gravitation, though they had not demonstrated it; but, whether light be thus subject to gravitation on classic standards or on those of the Einstein theory, it must be realized that in either event it must have mass. According to the classic view, it will then be subject to weight in the same way as any other body. According to the relativity theory, it must still have mass, since no demonstration or even discussion has been had by the Professor or any other relativists, so far as the writer knows, with respect to apparent motion of any entity not having mass. Human experience can have no contact with any such entity. Mass may be analyzable into energy, but it is still substance. Curving space may press mass into apparent motion, but not even a relativist can discuss its effect on spirit.

The second matter of importance mentioned by the Professor as resulting from his demonstration of light deflection by gravity is that the velocity of light can no longer be constant. A curvature of rays of light can only take place when the velocity varies with position. This disposes then of a fundamental assumption of the special theory; and the general theory cannot depend upon equations containing the constant, *c*. That constant has become a variable.

But, as above indicated, Einstein now proceeds to show, again by illustration, that we can no longer depend upon the dimensional magnitudes of those equations. The stability of our co-ordinates is destroyed by the conception of the space-time world. Our Euclidean mathematics no longer suffices. This serious objection is elucidated by Einstein by his famous illustration of the rotating disc.

He supposes an empty space in which there is a large, flat, circular disc rotating uniformly in its own plane, and on it one or more observers. In the distance may be imagined another observer on a reference body relatively at rest. The disc will, of course, be rotating uniformly from the point of view of the outside observer, but to those on it, its motion will be unknown. The disc observers, however, will be conscious, and the more so in proportion as they advance from its centre toward its periphery, of a force tending to pull them in a direction which, to the outside observer, will be tangential to the rotating motion. They will find by experiment that the strength of this force, unlike that of gravity, will vary in direct proportion to the distance from the point which the outside observer recognizes as the centre of the disc; and they will observe that the force will be present whatever the apparent intervention of other bodies on the disc. They will thus be able to attribute this force to a gravitational field, of a different kind, to be sure, from that with which we on earth are familiar, but, nevertheless, operating in the same general way. To the outside observer they will seem to be subject, not to the operation of a gravitational field, but to the centrifugal force due to the rotation. The disc observers will be quite justified in regarding themselves at rest and subject to this gravitational force. To the outside observer, the appearance of a force tending to push or pull them outward is a manifestation of their own inertia resisting the rotatory motion.

Our disc observers, however, are scientists and, knowing of the Einstein theory, are not disturbed by the fact that their gravitational field acts in so peculiar a fashion, because they are convinced that a general law of gravitation can be formulated whereby their own experiences may be transformed into those existent in other and different fields. With this idea in view, they attempt to make experiments in their own field, but discover that they can make no preliminary, harmonious observations. If identically constructed clocks be placed at the point which is the centre of the rotating disc, where there is no motion, and at various other points on the disc, where there are varying amounts of motion dependent on the distance from the centre, no two clocks will keep the same time. As the special theory has told us, time units are longer the greater the velocity.

According to Einstein, it is the same when the disc observers make space measurements in their world. If they make their first measurements on a line which turns out to be a diameter of the disc, since their measuring rods will, in this case, be perpendicular to the direction of motion, the result will be the same as though the measurement were made on the disc at rest. Selecting their measuring rod of a sufficient shortness to be applied to the measurement, as straight lines, of small arcs of the circumference, the rod, being this time applied in the direction of the motion, will, according to Einstein, shrink and therefore produce a larger measurement of the circumference than if the disc were at rest. Now, in our own world the circumference of a circle is always the same number of times the length of the diameter, approximately 3.14. For the purposes of our illustration, let us fix this figure at 3. If, therefore, the diameter of our disc at rest is 8,000 measurement units, the circumference will be 24,000. When the experiment is made, however, according to Professor Einstein, the circumference will measure a larger number, say, for example, 48,000, while the radius will still remain 8,000, so that the ratio of the circumference to the radius will now be 6 instead of 3. In our disc world Euclidean geometry is annihilated.

Now any one must be convinced that, under the theory of relativity, Euclidean geometry ceases to exist; and, since this is the point made by the Professor through the disc illustration, it may appear invidious to criticise the illustration as inept, but it is here attempted because of the curious and interesting results.

To the disc dweller, while the measuring rod shrinks, each arc of the circle to which it is applied will likewise shrink, unless there be something in the form or motion of a rotating disc which would interfere with this result. If the measured arcs shrink in the same proportion as the measuring rod, the rod, during its application to the entire circumference, must measure precisely the same number of units as if the disc were at rest, to wit, 24,000, so that, although there has been certainly a shrinkage, the ratio of the circumference to the diameter will remain as before.

But perhaps the conclusions are intended to be stated only from the point of view of the outside observer. This seems probable from the text, though there is some confusion on the point. We ought, therefore, to re-examine the matter from both viewpoints.

So far as the clocks are concerned, the outside observer will, of course, see that the time units of the various clocks on the disc differ from his own, and from each other according to their respective distances from the centre, because the velocity of the motion of a clock will, of course, be greater the farther it is from the centre. By a disc dweller one would expect that the same result would be reached, because, unless he should have located himself at the same position as that occupied by any observed clock, he will have a velocity different from that of the clock. On the other hand, the disc dweller is not conscious of his own motion or of that of any part of the disc. The velocity of rotation at the periphery may be enormously greater than at that inner circle where he may stand, but as all objects on the disc, himself and clocks included, maintain their relative positions, the motion for him simply does not exist. It is the same, unless he could occupy a mathematical point, if he were to stand at the centre, which is, only for that point, at rest. Everything rotates together, so that the rotation would be as unnoticeable as that of the earth. The interesting question, therefore, awaits solution, whether under the principle of relativity, he would be able to detect any difference in time units on the disc, since he would have no means of becoming conscious of any relative differences of velocity, other than the fact that the peculiar gravitational force of this particular field as experienced by himself varies with the distance he occupies outward from the centre. There is no relative motion, and the theory denies actual motion.

So far as concerns the measurement of the periphery, this is not the same case as others discussed in previous chapters. Here we do not have measurement, by standards of a relatively moving system, of lengths in a neutral zone, such as the measurement of the distance traversed by light in space, that is, not on the system. Here we have measurement by the disc dwellers on their own system, where the shrinkage is common both to measuring rods and the lengths measured. The outside observer will, of course, see both shrinkages, as compared with his own standards, but, whether he be deemed to make the measurements mentally at a distance with the perceived disc standards, that is to say, the measurements of shrinking arcs with shrinking rods, or, as in the text, to observe the number of times of the actual application of the rod by the disc dwellers, he will see that application, actually or mentally, around the periphery 24,000 times, and to the diameter 8,000 times.

While we have not on earth any molar velocity approaching that of light, we are able to give very high velocities to revolving wheels, which ought to be sufficient, if the periphery shrinkages actually occur, to make that phenomenon evident by physical experiment. On that basis, since there is no shrinkage in the spokes, which are perpendicular to the direction of motion, ought not there to be ultimately a collapse of the periphery? This we must leave the physicists to decide; certainly no such phenomenon has ever been observed. Other problems suggested by the disc illustration are now set forth.

Would the rotating disc, by reason of the shrinkage in its periphery which, according to the theory, must occur, whether its measured proportions remain the same or not, have ceased to have the form of a circle? To answer that problem requires more than mundane mathematics. From one point of view, it is a circle, from another, not. To picture what it really is requires more the insight of poetic imagination than that of mathematical reasoning.

If our disc cease to rotate and now be made to travel in a straight line in its own plane, one of its diameters being coincident with the direction of travel, this diameter will be shortened according to the theory, though not detectably so to a disc dweller because the measuring rod applied will be similarly shortened; and the diameter perpendicular to the first will remain the same. The arcs of the periphery measured in the same way as already described, while shortened in the direction of travel and proportionately in the intermediate directions between that of travel and the direction perpendicular to that of travel, will, by reason of the corresponding curvatures of the different quarters of the circle and the corresponding shrinkage of the rod, yield the same number of units. So they will also, to an outside observer, watching the applications. The Euclidean ratio of circumference to diameter will be normal. To the outside observer the disc will look flat in the direction of travel. Is it still a circle or not?

Suppose a material right angle triangle to be sent on a similar journey in a straight line, one of the perpendicular sides being coincident with the direction of travel. This side will be shortened, the side perpendicular to it will remain the same, the hypotenuse will be shortened, and we trust, as the result of the equations, proportionately, as otherwise the triangle must disintegrate as such. The measuring rod, for reasons already explained, will give the same length of each side as before the journey began. For an outside observer, the triangle will be shorter in the direction of travel. Is the triangle still such, and of the same dimensions?

The answer to these questions is, again, the point of view. These objects have their real existence in the space-time world. Their forms as we view them are our way of interpreting them. They are different according to the relative state of motion. The circle is, of course, not really a circle, or the triangle, a triangle. The real form of each is inconceivable.

The result of the disc discussion, whether the latter is necessary or not to its attainment, is, as we remarked at the beginning of the chapter, the clear fact that, in the extension of the special theory to the world as it actually exists, we can no longer rely on Euclidean geometry or on the co-ordinate quantities and systems which we are accustomed to use in mundane mathematics. The Lorentz transformation must, therefore, for most purposes be discarded. Its *c* has become a variable and its *x*s, *y*s, *z*s and *t*s have become impossible. Dimensions no longer exist.

In the development of a mathematics for the general theory, the Professor, therefore, would seem to be deprived not only of the instruments of classic calculation, but even of those of the special theory, for nothing remains to him of that theory for this use, except the conception of the space-time world with its attendant conception of the non-actuality of motion, to which has been added the principle of equivalence. We shall now endeavor to explain, in an elementary way, the methods by which the Professor attacked this problem.

He begins by the adoption of a system known as that of Gaussian co-ordinates, devised long ago by a non-Euclidean mathematician seeking a method for investigations in an imaginary continuum of more than three dimensions. A continuum, plane or three-dimensional, is clearly understood, from the common-sense standpoint, to be of such a nature that one can travel in it from point to point without jumps. Philosophically, as we have seen, such a thing is beyond understanding. It connotes continuous motion in eternity. The assumption of its phenomenal existence, however, works, producing mathematical results of a precise nature.

To explain Gauss's system of co-ordinates, let us begin by illustrating with an ordinary plane continuum. On this plane there are first supposed to be drawn a system of arbitrary curves, which may not, however, intersect each other. We may not say that they are parallel, because we must remember that when we extend our continuum into a four-dimensional one, the concept of "parallel" is no longer apposite. These curves are imagined to be drawn with infinitesimally minute intervals between them. In other words, we must imagine that an infinitely large number of curves may be so drawn and that each of them may be designated by a number for the purpose of location. In the same way, we imagine another system of curves drawn on the surface, fulfilling precisely the same conditions, but intersecting the system of curves first described. The location of any point in the plane is fixed by the designation of the respective numbers of two such intersecting curves.

If now, it is desired to determine the minute distance between two neighboring points on the surface, we imagine a line drawn between them and the further drawing, through both those end points, of the two curves in each case which locate them. If anyone will draw a diagram according to the above description, he will see that he will have formed a triangle. The curves of each system are, according to our ideas, parallel, and, for the minute distance involved, may be deemed straight. The triangle, therefore, will be a right angle triangle, of which the distance element between the two points is the hypotenuse; so that, according to the Pythagorean theorem, its square will be equal to the sum of the squares of the two minute arcs. If, however, the sides of the triangle are deemed curves, the resulting equation will require the multiplication of each square by a different potential which may be designated as g_1, g_2, etc.

This system is extended to continua of three or four or even more dimensions by simply adding further systems of curves running in the respective directions of the dimensions added. Thus, in a four-dimensional continuum each point will be located by four numbers designating the respective positions of the four curves whose intersection locates that point. The result is that the square of the distance between the two points, which is now no longer distance but interval, as we saw in Chapter VIII, is equal to a complicated expression, which may be simplified for our purposes to read somewhat as follows:

$$g_1 x^2 + g_2 y^2 + g_3 z^2 + g_4 t^2.$$

It will thus be seen that the Gaussian system is not one of co-ordinate dimensions at all, but, rather, of relationships. We saw in our elementary discussion of the mathematics of the space-time continuum in Chapter VIII that the sides of the triangle, other than the hypotenuse, were based on definite dimensions of co-ordinate differences. Here, the idea of any co-ordinate reference system is eliminated and all the so-called "distances" are conceived as minute intervals between "adjacent" points in the continuum. The Einstein idea is that the nature of the continuum, that is, of the geometric structure of the gravitational field, created by the energies in play, may be determined by the magnitudes of the g potentials, as we shall see later. The numbers by which the curves are designated in the Gaussian system are for purposes of location only, in much the same way that a house is located by its street and number.

From the foregoing, it is evident that the Gaussian system makes two assumptions. The first is that a non-Euclidean space-time field, although we can form no conception of it whatever, is still a continuum. The second is that minute portions of the field approximate Euclidean characteristics, so that the new mathematics may be built up on such a proposition as the Pythagorean theorem.

The Professor, without further explanation of the manner of use of the Gaussian system, now puts his general theory into final form. It will be remembered that the special theory posited the equivalence of all uniformly and rectilinearly moving systems for the formulation of the laws of nature. The first statement of the general theory posited the equivalence of all systems for such formulation, irrespective of their states of motion. We have seen that this statement was impossible of mathematical expression in such terms as those used in the Lorentz transformations. The use of classic co-ordinate systems had become impossible. Einstein, therefore, now makes his final statement of the general theory as follows:

> "All Gaussian co-ordinate systems are essentially equivalent for the formulation of the general laws of nature."

In classic mathematics, we are compelled to use a number of different kinds of co-ordinate systems, some, such as the system of polar co-ordinates and that of latitude and longitude, being designed more for the mathematics of curved lines and surfaces. The Cartesian method, which we have already described, was peculiarly applicable to the special theory and the Lorentz transformations. In the space-time world containing accelerated motions we are not able to make final or uncorrected use of any of these systems. The adoption, therefore, of the Gaussian system for such a world must mean that any appropriate system of co-ordinates may be used, or, in a theoretic sense, no system at all. The problem becomes one of determining the nature of the continuum, not by the use of dimensions linear or angular, but by relationships. It is a matter of comparison between one kind of continuum and another. The inquiry is in respect to their differences. In fact, after the description we have had of the Gaussian system and the above final formulation of the general theory, we are able to see, more clearly than before, that the task of the general theory is to enable us to formulate such a system of mathematics that, whatever the point of view of the observer, nature will be seen to be the same. The differences that appear, according to the condition of the observer, we must be enabled by its aid to smooth out.

The Professor explains this thought by telling us that the equations which express the general laws of nature pass into equations of the same form when by making use of the Lorentz transformation (in an appropriate world) we replace the space-time variables of one co-ordinate system by those of another; but that, according to the general theory, by application of arbitrary substitutions of the Gauss variables, the equations must pass over into equations of the same form, because every transformation, not only the Lorentz transformation, corresponds to the transition of one Gauss co-ordinate system into another. It is a matter of the different "curvature" of the different fields.

The man on the embankment saw the perpendicular flight of the raven as a perpendicular flight. The man on the moving train saw it as a flight on a slant. Either observer would have seen the phenomenon precisely as the other saw it, if he had moved himself from one co-ordinate system to the other, or had performed that operation mentally through the use, in this crude case, of classic mathematics. The outside observer, in another illustration explained above, saw a ray of light moving in a straight line, but to another observer, differently situated, the ray was moving in a curve. The thing would have been seen by either observer as the other had seen it, by the proper transformation of co-ordinates, and, for practical purposes, again by classic mathematics. According to the principle of equivalence, all these motional phenomena are at bottom the same; and with the high velocities involved in the deflection of light in a gravitational field, classic mathematics no longer avails even for a correct mundane picture. It becomes necessary to allow for the resulting space-time curvature. If we use the Gauss co-ordinate system, however, that is, if we dispense with dimensional quantities in the co-ordinates, we may be enabled to determine that curvature for particular fields, and thus, by the making of proper allowances, reconcile all points of view.

Einstein has devoted his energies to the elaboration of the proper mathematics, not with respect to all laws of nature, but with respect only to gravitation. The discovery of a law of gravitation, however, is in a sense a general theory, because gravitational acceleration is the controlling fact in the mechanism of our universe and because the discovered law results, if valid, in the demonstration that gravitation is not a force, but a geometry, the structural form of the particular space-time continuum in which it is manifested. This being true, we should have strong ground for the belief that all forces are only relative, expressing the mode of physical activity or energy; or, to put it differently, expressing the medium created by such energy for its own manifestation.

The simple equations that we have given at the end of Chapter VIII are subject to modification according to the co-ordinate system chosen and also according to the curvature of the surface in which the distance element is constituted. So long as our problem deals with two dimensions or three dimensions and with no curvature, the simple formula for the square of the distance element is $x^2 + y^2$ or $x^2 + y^2 + z^2$, changing, however, when using co-ordinate systems other than the Cartesian, or when dealing with curvature, into multiplications of the various quantities by different g values. When these mathematical expressions are extended to the four-dimensional continuum, the square of the distance element will take some such form as $g_1 x^2 + g_2 y^2 + g_3 z^2 + g_4 t^2$.

The problem which Einstein set himself was to determine how much of the quantities represented by the various gs was due to the use of the particular co-ordinate system and how much, in any Gaussian system, to the curvatures involved. In using this method he must, as already remarked, be making the working or hypothetic assumption that the inconceivable twists of the space-time continuum will, for minute quantities, be solved by the same mathematics as that which is used for the determination of curvatures in space. His ingenious work resulted in a determination of that part of the values of the gs which was caused by curvature only and, therefore, had no relationship to any particular, classic co-ordinate system. In other words, his determination was of these g values as expressive of the curvature differences of the respective regions in respect to which they are obtained.

The Newtonian law of gravitation, empirically ascertained, has been applied successfully only where ordinary velocities obtain. Einstein's mathematics enables him to determine the different curvatures of space-time continua according to the intensity of the particular gravitational field and the particular velocities involved. He is thus enabled to correct the Newtonian law, as applicable to the Newtonian region, but by a quantity so slight as not to be manifest in calculation except in one phenomenon where a relatively large velocity is involved, that connected with the perihelion of Mercury, which will be mentioned in the next chapter.

We have already taken this attempted explanation of the method of the Einsteinian mathematics farther than the Professor himself takes it in his popular treatise, and shall conclude the difficult subject with one further attempt to put its meaning into concrete form. If a mass were to exist alone and not moving in our sense, it would still be moving, by the fact of persistent existence, along the time dimension. If it now received actual motion in our sense, it would be active in the space-time continuum in a way that is interpretable by us as ordinary motion in space and time. Moreover, in the case supposed, our point of view would not be greatly in error. Space-time in such a case is said to be flat. But the particular kind of space-time is the offspring of those static and kinetic energies within the body, which reveal themselves to us in the form of motion in space and endurance in time. The apparent motion of such a mass, to us, is determined by the guidance, so to speak, which it receives by the particular structure of the space-time created.

If now another mass is introduced into the universe it will likewise create its own space-time or outlet, according to the energies within it, but the result of the two space-times thus created will be that of acceleration of the two bodies in a gravitational field. One may look on the "motions" of the two bodies as centrifugal or centripetal, inertial or gravitational, according to the point of view. They are the same. They depend upon the geometry of the medium; and that geometry or structure, the particular amount of "curvature," that is to say, the quantities of the *g*s, depend upon the mass, upon the energies involved. Newton's old problem of the possibility of gravitational action at a distance is eliminated.

If Einstein's alleged discovery of a new law of gravitation means that by an entirely new mathematics he has reached, for ordinary velocities, the same result as Newton empirically reached and empirically demonstrated, that is, the law of inverse squares, that would have a tremendous bearing upon the truth of his theory and, under certain conditions, might be conclusive. It rests with some great mathematical mind to analyze his processes and state whether he has deduced the law, and with a new precision, from the truth of precise space-time formulations, or whether he has taken out of his equations that from classic ideas which he has put into them, and, assuming the Newtonian law or its unknown classic basis as at least an approximation, merely added to it a supposed and minute correction. That correction is so trifling as to have proved incapable of test except in relation to the motion of the planet Mercury and the gravitational deflection of light. These matters will be discussed further in the next chapter.

CHAPTER XII

PROOFS OF THE THEORY

IN this chapter we shall attempt a brief account of the few data and experiments which have been advanced in support of the theory.

(1) THE FIZEAU EXPERIMENT. This was first performed nearly seventy-five years ago. Its object was to determine the velocity, in relation to a tube, of a ray of light moving through a liquid, in turn moving, in the same direction, through the tube. It might have been supposed that the answer would be the addition of the velocity of the liquid through the tube to the velocity of light in the liquid when stationary. The experiments, however, resulted in a smaller velocity. Fizeau discovered a formula whereby this result was approximated, requiring the multiplication of the addition of the two velocities by a fraction into the composition of which entered the index of refraction of the liquid, that is to say, the ratio of the velocity of light in vacuo to the velocity of light in the liquid. Einstein

states that he approximates the same result, but, in order to do so, he frankly subjects his formulae to a process which looks like unintentional forcing. He calls the replacements which he makes in Fizeau's equation, in order to produce his result, a process of approximation. The data and the differences to be worked out are minute and delicate. Moreover, he assumes, on the basis of his denial of the actuality of motion, that light must travel through a liquid in motion at the same velocity as if the latter were at rest. But we are ignorant of the method of passage of light through the interstices of a flowing liquid in relation to the molecular and atomic disturbances that may result if we adhere to the belief in the phenomenal reality of motion. The experimental result may be due to Einsteinian space-time, but it may likewise be due to physical refractions and interferences, not yet understood, connected with the passage of light through the moving liquid. At all events, Einstein's solution of this problem is far from convincing.

(2) SPECTRUM LINES. The gravitational field of the sun being vastly more intense than that of the earth, it will be evident that, if the theory be valid and we were able to compare

the action of two identically constructed clocks, one on the sun and one on the earth, it would be found that the former would move more slowly; its time units would be longer. This assumes that, there being no such thing as real motion, the potential of the gravitational field has the same effects as apparent velocity. Marvelous to say, we find in nature itself a means whereby we can compare the action of a sun clock with an identically constructed earth clock. Each atom of every incandescent element on the sun, disregarding as immaterial to the present discussion the electronic composition of the motion, vibrates at a particular rate, which ought to be the same as that of an atom of the same element when rendered incandescent on earth. Moreover, the solar rate of vibration, that is to say, the time interval of the solar ticks, so to speak, is transferred to the earth in the resulting light rays, unless transformed when they reach our space-time, which Einstein assumes not to be the case. The physical magnitudes of the sun being known, mathematicians are able to calculate the intensity of the solar gravitational field, and thus to ascertain its effect on the solar tick intervals under the Einstein formulæ. If these intervals turn out to be longer than those shown in the lesser gravitational field of the earth when the ticks of the same element, rendered incandescent on earth, are measured, the result will be some evidence of the truth of the Einstein theory. The comparison can be made by means of the respective spectra. Einstein has calculated the effect that, according to his theory, ought to manifest itself on the solar spectrum as compared with the terrestrial one. He has figured the exceedingly minute amount to which the spectral line of the element used should be displaced, as compared with the terrestrial spectrum, toward the red end of the solar spectrum, the end of longer waves and slower vibrations.

The same phenomenon must appear in the comparison of stellar spectra with terrestrial spectra, but, the stellar magnitudes not being known, precise displacements cannot be predicted. Now these displacements of particular lines of the solar and stellar spectra, as compared with the same lines on terrestrial spectra, do, in fact, exist, but the problem is rendered exceedingly complicated by the fact that other causes operate to produce similar results, such as the motion of earth and sun or earth and star toward or away from each other, the orbital motion of the earth and atmospheric refractions. The complex problem has not yet been solved, and it is believed that no one claims that the displacements observed are convincing proofs of the Einstein theory.

(3) THE PERTURBATION IN THE MOTION OF THE PLANET MERCURY. For a long time it has been known that, after due allowance for all the known factors, the motion of Mercury in its orbit about the sun shows a slight perturbation or variation from what it should be according to the calculations of the Newtonian celestial mechanics. This manifests itself in the travel of the orbit itself in its own plane, so that its major axis is swinging slowly about the sun in such a way that after a very long period of time the direction of this axis will be radically changed, the complete rotation, due to this unknown cause, taking thousands of centuries.

To put the matter in another way: At one end of the major axis is the perihelion or point of nearest approach of Mercury to the sun. As each revolution of the planet in its elliptical orbit is completed by its return to perihelion, it is found that the point of perihelion has been moved in the plane of the orbit beyond what might be expected from all the known elements, such as the disturbing attraction of the other planets; and to such a degree that, after a sufficient number of revolutions, from this unaccounted for disturbance alone, the direction of the ellipse will be noticeably changed. Leverrier published in 1859 his calculation that this additional change of position of the perihelion amounts to 38 seconds of arc per century. Newcomb in 1882 found this magnitude to be 43 seconds, and in 1895 published his final computation as 41.6 seconds.

Einstein claims that this phenomenon is inexplicable by Newtonian standards, but that his mathematical formulæ result in the computation of the perturbation at 43 seconds. This he regards as very strong proof for his theory. He either assumes that the Newcomb calculation of 1882, which is the same as his own, is the more accurate one, or regards the averages of the three different calculations to be close enough to his own to warrant his conclusion. Before his calculation can be regarded as conclusive, light is needed on facts as yet unknown, and certain elements in the problem must be most closely and analytically considered by specialists.

The orthodox astronomers are far from conceding that the variation is not solvable by the Newtonian mechanics if we knew all the factors. It has been calculated that millions of meteors daily come into collision with the earth's atmosphere; and the space of the solar system probably contains countless trillions of these little bodies. The natural expectation, based on the known laws of gravitation, as well as the evidence, points to denser accumulations of these bodies the nearer we approach the sun. Mercury is the planet of our system nearest the sun and great swarms of meteors on either side of its orbit could account for the whole, or a part of the movement in question.

A portion of this movement could also be accounted for by the fact that the sun is not a perfect sphere. It is an incandescent mass, largely gaseous, and the form of which is probably variable and slightly oblate at the poles. In the larger calculations of motions in the solar system, the various bodies are deemed spherical and the resulting calculations are, therefore, only approximate, though sufficient for most purposes. In a planet so near the sun, however, as Mercury, it may well be that a change in the gravitational calculations by reason of the solar equatorial bulge will account for a substantial part of the Mercurial variation.

Moreover, the Newtonian calculation of Mercury's perturbations leaves the one in question unaccounted for only on the hypothesis that this calculation is absolutely correct. It is, however, an extraordinarily difficult and complex calculation, assuming all the factors known, whereas it may well be that many are not known. Besides those already mentioned and entirely omitted from the calculation, it is only necessary to add that the calculation must assume the precise masses of the disturbing planets, while several of these are only approximately known.

Moreover, it is possible, even highly probable, that the disturbance may be due to the gravitational action of an undiscovered planet between Mercury and the sun. Every one will remember the wonderful method of the discovery of the planet Neptune, some seventy-five years ago. Similar perturbations had been noted in the movement of the planet Uranus. Using the Newtonian law, the existence, position and other particulars of a new planet, sufficient to account for these perturbations, were calculated; and Neptune was promptly discovered in the location predicted. The failure to discover the disturber of Mercury, even if it exist, is not surprising, considering its proximity to the sun. Mercury itself is so close to that orb as to render observation of it difficult.

It is that nearness and Mercury's corresponding relatively great velocity, besides the exceptional eccentricity of its orbit, which render more detectable the perturbation above described than in the case of the lower velocities and lesser orbital ellipticities of the other planets. But whether the detectable variance agrees with the Einsteinian formulæ is a matter for mathematical experts. There is always the danger, as in the case of the Fizeau experiment, of an unconscious forcing toward an already known value.

(4) THE DEFLECTION OF STARLIGHT BY THE SOLAR GRAVITATIONAL FIELD. This is the most famous of the proofs adduced in support of the Einstein theory and the most interesting, not only because of its cosmological implications, but because the astronomic observations of this phenomenon, which are believed by some to establish the theory, were made by great scientific expeditions in order to prove or disprove a prediction which Pro-

fessor Einstein courageously made several years before the observations were first made. We have seen that the Professor in his book (published in its present form in 1916) concluded, on the basis of his illustration of the man in the chest, that light rays will be curved in a gravitational field. He predicted, if a proper observation were made of the position of a star near the limb of the sun during a solar eclipse, when alone a star in such a position can be seen, and this observation then compared with the star's position in the absence of the sun, that the comparison would show a deflection of the firmamental position of the star observed on the limb of the sun, from that observed in the sun's absence; and he even predicted the precise quantity and direction of the deflection.

The observation was first made in 1919, and to the astonishment of the scientific world the deflection was established. As such a phenomenon had never before been observed, it is not surprising that the proved existence of the deflection in any degree whatever and in any direction, should have tremendously increased the prestige of the bold prophet and led perhaps to an overhasty acceptance of his views. It seems invidious to say that the deflection, though prophesied, may have been a coincidence only, that is, due to other than Einsteinian causes; that this may be again the case of a Columbus who, while endeavoring to reach the east by sailing west, discovered a new world, though not the one expected. And yet what Einstein, in the effort to prove his theory, may really have proved, may be the important fact, long suspected, that light has weight in the orthodox sense.

Any reader, though far from an expert astronomer or physicist, who will study the description of the apparatus used in these observations and the large margin of error possible by reason of defects therein, will readily comprehend that, in view of the required delicacy of measurement of the things observed and of the error allowances both for apparatus defects and other possible physical causes of the observed phenomena, the greatest caution in the analysis of the results is necessary. Some astronomers deny that the photographs of the eclipse observations, when compared with those taken of the same stars in the absence of the sun, show deflections approximating the amount or the direction predicted by Professor Einstein. An examination of the various tables of the deflections observed shows that many of them are far away from the quantities predicted. The quantity approximating the predicted one is obtained by averaging a selected few of the observations.

But even if the deflection be established, and of a quantity approximating that of Einstein, the scientific world, in the absence of overwhelming proof, will probably be slow to accept the cause as Einsteinian in its nature. The quantities involved are minute, and the assignment of causes that might produce the same result under the accepted mechanics of the universe too plausible. The amount of deflection caused by the refraction of the star rays by passage through the solar atmosphere is not capable of present computation owing to our present ignorance of the solar atmospheric strata. And why may not light have Newtonian weight? That it has such weight, has long been suspected by physicists.

To be sure, this means that light has mass, but our ignorance of its true nature is sufficiently dense not to require the rejection of that hypothesis. It will be remembered that Newton's original theory of light was the corpuscular one, involving the travel of minute particles of matter, which, of course, must have definite mass. The wave theory of light, which seems best fitted to solve most of its phenomena, may also imply mass. Light is supposed to be an electro-magnetic phenomenon. Its motion seems to manifest inertial properties. It certainly is the seat of energy, and if the ether theory be true, of ethereal energy. But, as we have seen, modern research tends to identify certain kinds of energy with that which is interpreted by us as mass. And if mass, why not weight?

It is not surprising that, if light has weight, that fact should long have eluded observation. At the surface of the earth the fall would be 16 feet in one second. But in that time the ray will have traveled 186,000 miles. How should a curvature represented by a fall of 16 feet in a centrifugal travel of one billion feet be noticeable? And how much more is the detection rendered impossible, when one considers the relative size of the earth as the field of observation and the cause of the pull. Its diameter is only a small fraction of light's travel in one second. A ray will have traveled far beyond us before the attraction could have much chance to operate or be observed.

To Einstein belongs the great credit of suggesting the method of observation at the surface of the sun, where both the stronger gravitational pull and greater solar diameter render more possible the detection of the light curvature. The fall there is many times the terrestrial 16 feet in the first second, and the great size of the sun gives the force a longer time to exercise its accelerating pull upon the fast passing light.

But it is claimed that the deflection of the starlight will be noticeably greater under the Einsteinan calculation than under that of Newton; and hence that the agreement of the deflection with the former calculation is a proof that the Einsteinian rather than the Newtonian formula is correct. For the ordinary velocities of the solar system, the two are in substantial accord. Differences are not detectable. The great velocity of light, however, is supposed so to affect the field as to render this difference quite detectable. How relatively minute the difference is may, nevertheless, be realized from the fact that, in order to account for the Mercury perturbation, the occasion, it is true, of a less difference, Newton's law of inverse squares need only be corrected by adding to the power 2 of the law a decimal beginning with six zeros. But Newton's law is based on observation only. Perhaps it is but an approximation. Though it has proved sufficient for all the important movements in the solar system, why may it not need a minute correction for more delicate observations? That would of course not prove the reality of space-time, but merely the necessity of a more precise law in dealing with such centrifugal velocities as that of light. A correction sufficient to account for the detected amount of star-light deflection would be too small to affect the calculations relating to ordinary planetary motion.

Now the relativity theory of course also gives mass to light. Gravitational as well as inertial motion, which under that theory are the same, can have to do with mass only. The difference is that according to classic views, which everyone recognizes are, in this respect, far from final, the light, if subject to gravitation, is pulled by a force; according to Einstein, the activity manifested by its apparent motion is guided in that manifestation by the structure of the medium created by that activity. The most that can be said to have been proved by the eclipse observations is that light may have weight and that the beautifully simple gravitational law of inverse squares may need a slight correction.

(5) THE DEDUCTION OF A PRECISE GRAVITATIONAL LAW FROM THE EINSTEINIAN PREMISES. In the foregoing discussion of light deflection, the question mentioned at the end of the last chapter must have recurred to the reader's mind, whether Einstein has deduced, solely from original and non-Euclidean premises, a precise and independent law of gravitation, valid for all conditions, of which Newton's is only an approximation for given and usual conditions. If he has, that surely would be convincing proof of the soundness of his theory. We are not aware that anyone has advanced such a proof, but, in view of statements of Einstein himself, it surely should prove of the greatest interest to have from able mathematical analysts the answer to this question and the related ones below suggested.

Einstein's text seems to indicate a negative answer. For example, in the appendix, on referring to the deflection of light, he says:

> "According to the theory half of this deflection is produced by the Newtonian field of attraction of the sun and the other half by the geometrical modification ('curvature') of space caused by the sun."

If this quotation is to be taken literally, it means that the Professor has calculated the precise amount of the solar pull on light under Newton's law and then added to it a correction under his own. In this view the meaning would be that Newton's law, based on or confirmed as a theory by empiric observation of the apparent gravitational pull on relatively slow-moving masses, has accounted for only half the deflection and failed to account for the other half, which, now only, we are able to account for by reason of the additional twist or g value given to the medium in which the phenomenon takes place by the high velocity of light in space. One thing that one wants to know is whether this idea of a mere addition to the Newtonian law is a correct one, or whether the Einsteinian law yields the final answer, of which the Newtonian is but a limiting part.

The nearest approach to a discussion of these questions in the Einstein treatise is the following:

> "If we confine the application of the theory to the case where the gravitational fields can be regarded as being weak, and in which all masses move with respect to the co-ordinate system with velocities which are small compared with the velocity of light, we then obtain as a first approximation the Newtonian theory. Thus the latter theory is obtained here without any particular assumption, whereas Newton had to introduce the hypothesis that the

force of attraction between mutually attracting material points is inversely proportional to the square of the distance between them. If we increase the accuracy of the calculation, deviations from the theory of Newton make their appearance, practically all of which must nevertheless escape the test of observation owing to their smallness."

Now it may be that the Professor's meaning is that his own law will independently produce the total amount of the deflection and that, after crediting the amount of the solar pull on light, under Newton's law, to the element in Einstein's equations which represents space-time curvature, he has given the benefit of the remainder to the expression in those equations which is deemed to represent the curvature of the space element alone, due to the great velocity.

If this be true, the implication is that the Einsteinian law is a complete and final law of gravitation which includes Newton's as an excellent approximation for ordinary velocities. Even then, however, further questions remain to be answered. Newton's law, as we have frequently remarked, is unfortunately empiric only. From the time of its discovery the hope must have been entertained that some great thinker would eventually discover, on

Euclidean premises, why bodies should attract each other in the way apparently established by it. That they do so has been repeatedly demonstrated in the solution of the motions of the planetary bodies. May not the reason lie deep within the truths of classic mathematics? May not Einstein, working as he does on the basis of classic mathematics, notably the Pythagorean theorem, have reached, by inclusion in his computations, the underlying principle of the empiric law? And, in that event, may not his minute correction to that law, assuming its validity, be such a correction as results from the establishment of a law on a scientific, as distinguished from an empiric, basis? Is Einstein's law the exclusive product of his space-time and other assumptions, or is it the offspring of classic and Euclidean mathematics, which can live and thrive in a Newtonian world? Finally, are results approximating the Newtonian drawn from the Einstein equations because of the introduction into them of Euclidean and Newtonian premises? Such questions as these, better formulated than is possible for the present non-mathematical writer, should be answered by those qualified to do so.

CHAPTER XIII

EINSTEIN'S CONCEPTION OF THE UNIVERSE—CONCLUSION

THINKING men have, of course, always displayed the keenest interest in the creation of some kind of logical and connected conception of the universe as a whole. It is not surprising that their speculations on this subject have been to the highest degree unsatisfactory, in view of the great ignorance which they still have of many basic facts.

Since the Newtonian physics implies a universal gravitation, a common conception is that of a boundless space in which the existing matter, by reason of the gravitation of its parts, tends to condense itself, thus forming an island of material worlds floating in an infinite ocean of space. Some thinkers, including Professor Einstein, consider such a conception unsatisfactory. In the first place, the constant radiation of light and other forms of energy from the central mass into the boundless surrounding ether, never to return, means a perpetual impoverishment of the finite material universe, which, in view of the laws of conservation of matter and energy, must ultimately result in extinction and death. Then again, in spite of the relative enormity of the great masses of suns and worlds and nebulæ thus conglomerated under the theory, their mass, in the infinity surrounding them, becomes as nothing, and their average density, zero. Moreover, the condensation of the great suns into relative nearness means an enormous increase of the intensity of the gravitational field in the outlying regions, with velocities of unheard of values. But, perhaps, if the Einsteinian conceptions be true, these tremendous velocities might be correctional of some of the supposed incongruities of such a universe, inasmuch as they would tend, as the velocity of light was approached, towards the disappearance of the bodies subjected to them.

On the other hand, to assume that the Newtonian law of gravitation does not apply to the vast distances of the sidereal universe, is to assume that matter may be everywhere present in the infinite depths of space. That means that the average density of matter might be constant everywhere even to infinity and without the production of unduly great gravitational fields. The objection to this conception is that it is without any foundation other than baseless speculation. We know, or at least have reason to believe, that not only the planets and satellites of the solar system are in motion, but that that system as a whole is in motion, and that the millions of stars, each with its own attendant system, are in motion. We are entirely ignorant, however, of the nature of these motions and whether they obey the gravitational law.

These speculations, too, lose sight of our complete ignorance of the ether and its extent. No one knows whether the medium that carries to us the starlight may not be limited. If it is, our sidereal universe may be one of an infinite number of such universes, separated by enormous voids. Across those great vacancies no light can pass, probably also no gravitational force. The ethereal waves, great and small, will find an impassable boundary. But the universes and the containing space may still be infinite, though we must of necessity remain forever in ignorance of all save our own.

The noble and ambitious spirit of man, in attempting to soar to the heights of these great questions, is admirable, but the criticism is justified that the attempt is hopeless until he has a firmer bottom of fact from which to spread his pinions. For man in his present state of knowledge to attempt an integral construction of the contents and form and laws of the boundless universe in its stellar heights and electronic depths, in its present spatial and temporal home, or in its non-spatial and eternal reality, seems as absurd as for an ant to attempt to form a conception of the radio. He should rather share the feeling of the great Newton who, despite his relatively gigantic conquests of truth, said at the end:

> "I do not know what I may appear to the world, but to myself, I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

Professor Einstein believes, like Newton, that progress can be made, though slowly, and suggests that his theory enables us to form a more satisfactory conception of the universe as one containing matter of approximately average density everywhere. His conception is the difficult one of a quasi-spherical space-time bounded and yet limitless, determined in its nature everywhere by the matter present. In order to prepare us for such a conception, he borrows from the non-Euclidean mathematicians the following illustration.

He asks us to suppose a race of flat, two-dimensional beings living in the surface of what is in fact a sphere, though in them will be no possible conception of that fact. He gives these beings intelligence and the power of motion, but motion only in their surface, from which they cannot in any way move in an upward or downward direction. Now one may imagine them as flat as one pleases, as flat as the thinnest piece of gold leaf that ever was hammered, and yet it is impossible to conceive that they could be without conception of their own thickness, however minute. Moreover, the fact that these beings move in their surface must, of necessity, give them a conception of something beneath them and thus of a third dimension.

Overlooking these considerations, however, we may perhaps glimpse one of the ideas that the Professor is attempting to convey, to wit, that to these creatures a third dimension might be as incomprehensible and its discovery as marvelous as to us a fourth or any other dimension beyond the three we know. For sufficiently small portions of the surface of their world, the space of these beings would be Euclidean, because on such portions straight lines can be laid out and Euclidean measurements taken. If, however, they should attempt to measure larger portions of their space, they would, of course, find that their supposedly straight lines would not be straight lines, would not enclose triangles the sum of the angles of which is two right angles, and that their circles would not be such that the circumference would be 3.14 times the diameter.

At this point in his description the Professor, probably to show us by analogy what our own capacities might be with reference to a four-dimensional space, intimates that these flat creatures, by the exercise of reason, could discover the third dimension, reduce the confused geometry of their surface to a Euclidean geometry and even determine the shape and size of the sphere upon the surface of which they live. This they would do by finding, in the way presently indicated, that the straight lines of their surface are really curves and by theorizing concerning the Euclidean values which lie in the bounding planes of the larger and larger spherical segments mentally created.

If these creatures, from any point on their surface, draw lines of equal length which we, but not they, know to be curves, and run a line through the end of these equal radii, that line will be for them a non-Euclidean circle since its circumference will not be 3.14 times the length of twice the supposed straight, but really curved radius. These circles will become larger and larger as they draw the radii with increasing lengths, until the circumference of their universe is reached, and from then on, as the radii are still extended further and further, the resultant circles will become smaller and smaller, until zero is reached, the area enclosed during the process, finally constituting their whole world area. The Euclidean laws of plane geometry discovered by them as applicable to exceedingly minute regions of their surface will, thus, to their mystification, not apply to larger portions, but in the end these investigations will enable them to deduce the third dimension and the sphericity of their universe. The Professor finds the great charm of this illustration in the fact that, as the inhabitants of the surface, before their discovery of the third dimension, travel over and measure every portion of it, they will find it to be finite and yet, in his opinion, without limits.

Now, what is meant here by the discovery of the limitlessness of this surface, the writer finds himself unable to comprehend, unless it lie in the fact that travel will sooner or later bring the travelers back to the starting point. Until the flat people are able to reach the conception of a Euclidean sphere, their conception will, so far the writer is able to carry his imagination into their minds, be one of agnosticism and not of limitlessness. That is to say, if the meaning has reference to the constant return, by travelers who follow their noses, to a recognizable starting point, that fact will not produce the concept of limitlessness, but will merely prove utterly incomprehensible to beings without idea of dimensions outside of a surface. If, however, the meaning be that our flat creatures regard small portions of their world as planes, and in imagination project these planes into space forever, then, of course, in that sense their universe will have no limits. But such a meaning is beyond the premise, since we started with beings who know their surface only, and, being ignorant of a third dimension, cannot mentally project a plane as tangential to the surface, really curved, but not so to them. They know only a plane surface mentally extensible forever, but practically not so, and of geometric properties apparently differing according to the size of the field examined.

If, however, our flat travelers determine scientifically from these peculiarities that they are living on a sphere, they will have no belief in the limitlessness of their surface, because with the knowledge of the sphericity they obtain also the knowledge of its limits. To be sure, at the same time they ought to obtain also at least a dim conception of a third dimension up as well as down, that is, of infinity surrounding the sphere. Thus the supposed case fails, because the sphere will no longer be their universe.

As we have before remarked, one object of this illustration is to show that, if there could conceivably be a two-dimensional creature, his difficulty in realizing a universe like ours might be as great as ours in realizing a universe like Einstein's. But the illustration has another object. It is intended to give us some sort of a conception of the Einsteinian universe. We have seen that when these two-dimensional creatures draw equal radii from a point in their surface and connect the ends, the resulting line is the circumference of a circle, which, however, to them will not have the relationship to the radius that a Euclidean circumference has, since the radius in that case is not a straight but a curved line. We have also seen that, if these radii be extended more and more, the resulting circles will be larger and larger until the circumference of the sphere has been reached, and that thereafter they will diminish in size, until finally, when the antipodes is reached, the circle will constitute a point. It is such investigations as these which give these creatures some sort of a surprised conception of the third dimension and of the real nature of their world. In the same way, the Professor desires us to imagine our universe similarly curved in a fourth dimension. His idea is that, if it were possible for us to make a similar experiment, we should be similarly surprised by finding that after our radii had reached the largest circle of our spherical universe we should find the circumferences connecting their ends as they were extended further and further, also gradually diminishing to a point, thus realizing the vague conception that, as certain limits are reached, all motion is bent back upon itself in some inconceivable way.

Thus the Professor endeavors to have us perceive how our universe may be finite, though unlimited. He desires us to conceive of it as quasi-spherical and thus bounded by the spherical surface. He seems to mean by this that, since our idea of an infinite space is born of our experience of the ceaseless potentiality of motion, we can conceive of a limit to space, that is a bounding surface to a spherical space, by merely denying the potentiality of motion beyond the limit set. Any line of force at that limit must twist back upon itself. Velocities there must cease or curve inexplicably backward. In such a space there may, in fact must, be matter everywhere, since it is the energy constitutive of or resident in matter that makes the space and the particular kind of space that is expressive of the particular amount of energy.

The idea of the limitlessness of this spherical universe, then, is predicated upon the non-Euclidean nature of the closed space. We are to attempt to conceive of this space as one of warps, twists, curves, contortions, which thus logically may make room, even in its limits, for all quantities of matter. A fold in space may secrete a world, and were we not forbidden to think of space in our usual way, we might perhaps now say that two or more things can occupy the same space at the same time. This attempted conception of a finite space twisted without limits will not, however, aid our difficulty in the conception of limitlessness; because there is really no difference between a space infinite in an outward direction and a space limitless in an inward direction. In the present nature of our minds, it is impossible for us to conceive infinite space at all, although we are bound to assume its existence rather than one of limits, because, as we saw in an earlier chapter, the latter is, so to speak, more inconceivable than the former. Nevertheless, Professor Einstein's solution merely adds this latter and greater inconceivability to the outer portion of his universe while also adding the former inconceivability to the interior portion. To this he gives unimaginable twists and turns, making them the potential seat of infinite phenomena, although their labyrinthean depths may always return an imaginary traveler to the starting point.

We have now reached the end of our study of Einstein's conceptions. Their origin, in the last analysis, is evidently the ancient philosophic relativity, first developed into its modern form by the great English thinkers, then by the German philosopher, Kant, and later by Spencer. This is, in brief, that all knowledge is relative, that neither matter nor motion, nor, indeed, anything can be known in itself, or otherwise than in the relative form of our apprehension, and that space and time, while they are mental forms, that is, the methods of mental reaction to phenomena of matter in molar motion, may, nevertheless, relatively express unknowable, external modes of reality. Einstein attempts to examine these modes. Science before him had reached its present state, denying or at least ignoring that possibility, by assuming space and time to exist as realities, without warp or twist, and in the same way as they are assumed, in order to live, by all men and all beasts.

We are living in an age of intellectual restlessness. Economics, government, social organization, psychology, art and now even physical science,—everything has been set whirling in the mental maelstrom. New cults of the moment are offered by the thoughtless as substitutes for age-old institutions and concepts, whose uprooting they demand, though planted and developed in the soil of all races alike. Ill considered revolution is mistaken for progress, while respect is scant for that steady and progressive evolution which, to more prudent and perhaps less selfish minds, has seemed so fruitful a method in relation to the complexities of our far from simple world. It is, therefore, at the present time, of more than usual importance, while it would be folly to close the eye of science to any new pronouncement, however radical, that its merits should be subjected to the closest scrutiny of those whose heads are still cool and whose feet are firmly planted. And this scrutiny must be the more thorough in proportion to the radical nature of the new proposition.

The Einstein theory may be the first trumpet call of genius to a new science and philosophy destined to unravel for ambitious man the most interesting secrets of his mysterious surroundings. But, on the other hand, its denial of such fundamentals as the phenomenal realities of space, time and motion renders it subject, if not to suspicion, to the most rigid investigation in relation to its origin, nature, methods and proofs. Perhaps it is a successful assault upon the hitherto impenetrable citadel of the ultimate. If so, the storming party must encounter the disappointment of discovering that the citadel was not as strong as supposed, was not an ultimate at all.

On the other hand, it may well be that just as to some minds, on the religious side, Christian Science is, in its annihilation of matter, an ill-considered perversion of the modern metaphysics, so, on the scientific side, the Einstein theory is similarly an illegitimate outgrowth of the same metaphysics; that is to say, a mistaken scientific effort to pierce the veil of the Absolute by transmuting in some way into physical expression the ignorance concealed in our knowledge of the fact that time, space, matter and motion are known to us only relatively. If this be true, the illegitimate attempt is made by denying, except in an inconceivably twisted form, that which hitherto has been the basis of all science, the reality of the external world in time and space; and it ends pitifully in the assumed establishment of relativities only, by denying the very Absolute it was meant to unravel.

For while the space-time world is intended, in a sense, as a conception of the absolute, it needs little reflection to discover that it is not really such. We have seen, in Chapter IX, that it is probably necessary to concede the possibility that space, time and motion are not ultimate phenomena, but the forms of our mental reaction, in our condition, to a deeper lying ultimate, the space-time continuum. The classic, philosophic assumption of such an ultimate as noumenal is inhibited by the relativity theory, since it posits the possibility of phenomenal discovery of the nature of that ultimate through its manifestation in the only apposite terms yet known to us, to wit, space, time and motion. In other words, the Einsteinian claim is that the space-time continuum is a new phenomenal ultimate which reveals itself to us, according to our point of view, in those dimensional and time relativities which enable us mathematically to define its absolute or final, if phenomenal, reality. If the continuum were really noumenal, certainly not even a relativist would be heard to say that any mathematical discovery thereof would be possible. But if the continuum be phenomenal, there is no reason for regarding it as absolute or even an ultimate. If things apparently so final in their nature as space, time and motion are really not phenomenal ultimates, there is no logical reason whatever to suppose that the space-time continuum is such an ultimate. And if it be not, then underlying it there must be a deeper lying ultimate for future generations to discover, and so on *ad infinitum.* Thus, the Einstein theory becomes a pragmatic philosophy, a philosophy of relativities, promising eternal disappointment to the hopes of those eager to discover at least an ultimate phenomenal truth in things.

If it should be ultimately determined that the theory embodies, then, only fantastic conceptions of the universe, unfounded in fact and born of a too intense, though highly skilled, concentration on the abstractions of mathematics, it must be admitted, nevertheless, that they have been the interesting conceptions of an ingenious mind. In fact, if the theory be exploded in its entirety, it will still have fulfilled some useful functions, among which the writer ventures to enumerate the following:

(1) The study and analysis of the theory affords excellent mental exercise.

(2) The Einsteinian mode of thought acts as a necessary caution to thinkers, that those axioms which lie at the basis of all our science are the product of an experience, which, though relatively vast, is, in comparison with

the problems to be solved, exceedingly minute. Our mechanical axioms are largely based on our experience of molar motion. Our reaction to molecular motions has hitherto been in such non-spatial forms as sound, taste, smell, heat and unassociated vision. Perhaps, therefore, our habits of thought and our conceptions will need correction as science penetrates deeper and deeper into the mysteries of the electronic and ethereal world.

(3) The study of Einstein operates to remind us that the inconceivables of one age may become conceivables in another, and that it yet remains to be demonstrated that even those notions which appear to us most fundamental and final may not be analyzed in succeeding ages into deeper lying realities; that the human mind may not develop further not only in the magnitude but in the variety of its senses and powers.

(4) The Einstein theory has directed general attention to the recent investigations of the atom and the ether. The suggestion is hazarded that future study should be profoundly concentrated upon these subjects, upon the deep mysteries of the atom's structure and its relation to the mother ether, upon the nature of gravitation and also its relation to

the ether, in fact, upon all those subjects indicated toward the end of Chapter IX.

Who can say that the outcome of these investigations may not be an explanation, in harmony with classic standards, of such results as those of the Michelson experiments, if those results be valid, and in terms not of an inconceivable four-dimensional continuum, but of an all-pervading medium, which, while it has not all the characteristics of matter, is yet the source from which matter springs? Before attempting, except on overwhelming evidence, inconceivable theories which set at naught the most fundamental and age-old conceptions of the race, would it not be better, first at least, to attempt the solution of those recent discoveries which have surprised our science, by a more profound penetration into the nature of the ethereal medium whose existence has been indicated to us in so many phenomena? An ethereal twist or vortex or other mode of activity is within the domain of classic science and may afford more fruitful study than an inconceivable, twisted emptiness. Perhaps the universe may be ultimately analyzable into a fundamental energy, that of the ether. One product of this mother energy, matter, we know a little about. At least, recent science has approached very closely toward reducing it to a basic energy. But of that basic energy we know next to nothing. Does not the immediate future of our science lie in the exploration of that vast field?

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